High Productivity Computational Finance on GPUs

GPU Technology Conference
March 15, 2012
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Aon Benfield

- **Aon Benfield**, a division of Aon plc (NYSE: AON), is the world’s leading reinsurance intermediary and full-service capital advisor. We empower our clients to better understand, manage and transfer risk through innovative solutions and personalized access to all forms of global reinsurance capital across treaty, facultative and capital markets. As a trusted advocate, we deliver local reach to the world’s markets, and an unparalleled investment in innovative analytics. With more than 80 offices in 50 countries, our worldwide client base has access to the broadest portfolio of integrated capital solutions and services.

- **Aon Benfield Analytics**
  - Aon Benfield Analytics offers clients industry-leading catastrophe management, actuarial, rating agency advisory and risk and capital strategy expertise.
  - Sample risk analytics products
    - PathWise
    - ReMetrica
    - ImpactOnDemand
    - CatScore
Annuity Solutions Group

Hedge Program Management | Advisory | Software | Consulting

**Asset Management Services**
- Industry leading software and infrastructure and expertise means improved risk management, lower costs and better hedge program results
- End-to-end, real-time, transparent hedge program management

**Advisory**
- Unique ability to work with clients to review and improve candidate investment bank structured solutions
- Unique ability to work with clients to forecast, design and implement dynamic hedging solutions to jointly manage economic, financial reporting and capital risks

**PathWise™ Platform**
- The fastest, most scalable, and integrated high performance computing based variable annuity risk management platform in the industry
- Platform includes tools for hedging, pricing, and the calculation and forecasting of capital and reserves
- Guaranteed run-times and performance

**Consulting Services**
- Analysis of reinsurance and investment banking solutions
- VA Liability and hedge asset stochastic on stochastic modeling
- Performance Attribution, Grouping, Fund Mapping, etc
- Hedge strategy development and testing

- 40+ years of modeling, derivatives trading and risk management experience
- A tightly knit, uniquely knowledgeable team combing financial, software, high performance computing and insurance and capital markets expertise
Industry Overview

- Life Insurance companies sell Retirement Savings products to individual policyholders

- Since the early 1990’s, these products have evolved into complex Investment Guarantees that protect policyholders against three main types of risks:
  - Mortality risk (risk of death)
  - Longevity risk (risk of outliving retirement funds)
  - Investment risk (risk of financial losses)

- Variable Annuity assets in North America currently exceed $1.5 trillion.

- Examples of popular products by region:
  - Europe
    - With Profits (UK)
    - Equity Indexed Annuities (EIAs)
  - North America, Japan, South Korea
    - Variable Annuities (VAs)
Industry Overview

Highest Annual AV GMAB
Provides guarantee based on highest annual account value paid at end of 30 years
Industry Overview

- Insurance companies must **hedge** the systematic or non-diversifiable market risks associated with these products.
### Industry Overview

- **Common hedging instruments**

<table>
<thead>
<tr>
<th>Risks</th>
<th>Equity Futures</th>
<th>Interest Rate Swaps</th>
<th>Variance Swaps</th>
<th>Vanilla Options</th>
<th>Hybrid Options</th>
<th>Lookback Options</th>
<th>Structured Hedge</th>
<th>Reinsurance</th>
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</table>
Industry Computational Challenges
Industry Computational Challenges

- **Business end-users focus**
  - Users are Quantitative Analysts, Actuaries, Traders, Risk Managers, etc
  - The right tools must focus on the end-user requirements

- **Business logic and systems code must be continually adapted to changes**
  - Changing models, financial products, market conditions, and regulatory requirements
  - Changing technologies (Multi-Core, Cell Broadband Engine, GPUs, etc)

- **High Computational Throughput is required**
  - Large-scale real-time Monte Carlo simulations (Support Hedging Programs)
  - Nested simulations (Hedging Back Testing, Capital, Valuation)
  - High end-user productivity (not waiting for huge runs to complete)

- **Mission Critical Operations**
  - The intended use of such systems is mission critical
  - System failures or bugs can be catastrophic for business users
  - Automation and auditability are very important issues
Industry Computational Challenges

- **Business end-users focus**
  - Actuarial and quantitative financial analytics are extremely complex. As example, we consider the model specifications for one of the simplest stochastic models that we use in PathWise:

\[
\begin{align*}
\frac{dS_t}{S_t} &= r_t \, dt + \sqrt{\nu_t} \, dW^S_1 + \gamma \, dW^S_2, \\
\, dv_t &= \alpha(\theta(t) - v_t) \, dt + \eta(t) \sqrt{\nu_t} \, dW^v, \\
\, dr_t &= \beta(\phi(t) - r_t) \, dt + \sigma(t) \, dW^r,
\end{align*}
\]

These types of model specifications are not easily understood by traditional programmers or IT business analysts. Understandably, business end-users prefer to implement models themselves (e.g., using Excel and VBA).
Industry Computational Challenges

- Business logic and systems code must be continually adapted to changes
  - Change is constant
    - Financial modeling innovation
    - Financial products innovation
    - Evolving market conditions
    - Changing regulatory requirements
    - Technological innovation

- Traditional approaches
  - Enterprise IT systems slow to adapt
  - Shadow IT systems fill the gaps - patchwork of end-user developed, manually operated spreadsheets (potentially thousands of interlinked spreadsheets)
  - Slow, costly, error-prone

"There it is! I've isolated the origin of the firm's demise."
Industry Computational Challenges

- **High Computational Throughput is required**
  - GPU grid and/or cloud is an excellent solution
  - But GPU grid / cloud is difficult to program
  - Teams of end-users programming GPUs in low-level languages such as CUDA or C++ is suboptimal
  - Huge complexity added by attempting to scale to distributed systems (clusters, cloud)
  - General purpose, high level languages
    - Helpful only to a point
    - Development effort is not significantly reduced (end-users must still somehow architect and implement a highly complex software system)
    - Severe limitations and performance bottlenecks may be inadvertently introduced
Industry Computational Challenges

- **Mission Critical Operations**
  - **Requirements**
    - Complex business data-flow management
    - Job scheduling
    - Fault tolerance / failover
    - Operational workflows
    - Reporting presentation layers
    - Audit trails
    - Monitoring and Error Reporting
  - Not just about implementing CUDA kernels
### Industry Computational Challenges

#### Application Stack

<table>
<thead>
<tr>
<th>Who</th>
<th>What</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>End-users</strong></td>
<td><strong>Presentation Layer</strong></td>
<td>Queries, reports, visualization, real-time dashboards</td>
</tr>
<tr>
<td></td>
<td><strong>Business Logic</strong></td>
<td>Data structures, models, business rules</td>
</tr>
<tr>
<td><strong>Software Developers</strong></td>
<td><strong>Application support</strong></td>
<td>Training, documentation, troubleshooting</td>
</tr>
<tr>
<td></td>
<td><strong>Application deployment</strong></td>
<td>Application updates, configuration, version management</td>
</tr>
<tr>
<td></td>
<td><strong>Application development</strong></td>
<td>Frameworks, GUI, system-level code, CUDA kernels</td>
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<tr>
<td></td>
<td><strong>Middleware</strong></td>
<td>Grid middleware, messaging, web services</td>
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<tr>
<td></td>
<td><strong>Databases</strong></td>
<td>SQL, NoSQL (MapReduce), etc</td>
</tr>
<tr>
<td><strong>Engineers / IT Admins</strong></td>
<td><strong>Infrastructure operations</strong></td>
<td>Security, systems monitoring, maintenance</td>
</tr>
<tr>
<td></td>
<td><strong>Operating Systems</strong></td>
<td>Linux, Windows</td>
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<tr>
<td></td>
<td><strong>Virtual Machines</strong></td>
<td>Citrix, VMWare, Amazon Elastic Compute Cloud</td>
</tr>
<tr>
<td></td>
<td><strong>Servers</strong></td>
<td>Rack-mount (1-4U), blade server, proprietary rack</td>
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<tr>
<td></td>
<td><strong>Processors</strong></td>
<td>CPU, GPU, FPGA, Hybrid, ASIC</td>
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<tr>
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<td><strong>Storage</strong></td>
<td>SAN, fileservers, SSDs</td>
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<td></td>
<td><strong>Network</strong></td>
<td>1GigE, 10GigE, InfiniBand</td>
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<tr>
<td></td>
<td><strong>Data Center</strong></td>
<td>On-premise, co-location, cloud, garage</td>
</tr>
</tbody>
</table>
## Industry Computational Challenges

<table>
<thead>
<tr>
<th>Layer</th>
<th>In-House App Development</th>
<th>Outsourced Blackbox Solution</th>
<th>PathWise SaaS option</th>
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</thead>
<tbody>
<tr>
<td><strong>End-users</strong></td>
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<tr>
<td>Presentation Layer</td>
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<tr>
<td>Data Center</td>
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</tbody>
</table>

### Legend
- ✓ Implemented by end-users (good)
- ✓ Implemented by end-users (bad)
- × Not implemented by end-users (good)
- × Not implemented by end-users (bad)
Industry Computational Challenges

- **In-house software development (roll your own)**
  - High risk project for organizations that do not have extensive expertise in commercial software development
    - Costly and time consuming even for experts
  - Limited functionality and innovation roadmap
    - High-end solutions, such as nested Monte Carlo simulation and high productive end-user tools, remain out-of-reach
  - Long-term costs of maintaining such systems, under continual business change, are often underestimated

- **Software development outsourcing**
  - Difficulty understanding business requirements and scope
  - Language, cultural, and geographical barriers
  - High complex requirements (e.g. quantitative finance, scientific codes, etc) are not easily understood by programmers
  - Quality Assurance, Support and Security issues
Industry Computational Challenges

Example HPC Solution Trade-Offs

- **Flexibility** – ability to rapidly make changes
- **Specialization** – code specialized to specific hardware
- **Performance** – run-time performance of the solution
- **Reliability** – probable number of bugs in a large system

Size of bubble indicates cost (in terms of time and money) of solution
PathWise Platform
PathWise Platform

- **PathWise Solution**
  - **Domain Specific Visual Programming Language**
    - Generates C++/CUDA code (double and single precision)
    - Generates middleware interfaces
    - Spreadsheet-like interface for entering computation logic
    - SVN integration
  - **Data Management Solution**
    - Message-oriented persistent storage system
    - Specialized for dealing with numerical / financial data
  - **Python Integration**
    - High-level APIs for steering grid computations, accessing business data, visualizing results, and creating reports
  - **Platform-as-a-Service offering and Cloud Integration**
    - End-to-End management solution
    - Amazon EC2 integration
PathWise Platform

- End-user tools for High Productivity Computing
PathWise Modeling Studio

- Create a new model
PathWise Modeling Studio

- Define input data structures (customized NumPy data structures)
PathWise Modeling Studio

- Setup Random Number Generator options
PathWise Modeling Studio

- Import and configure Model Libraries (e.g. pre-built Economic Scenario Generators)
PathWise Modeling Studio

- Calculate number of time-steps to simulate
PathWise Modeling Studio

- Define simulation columns and formulas
PathWise Modeling Studio

- Encapsulate re-usable logic in UDFs and UDF libraries

```c
void cholsky(real[N,N] matrix)
{
  if N = 1 then
    $result[0,0] = matrix[0,0]
  end
  for i = 0 to N-1
    for j = i to N-1
      real sum = matrix[i,j]
      for k = 0 to i-1
        sum = sum - $result[i,k] * $result[j,k]
      end
      if i == j then
        $result[i,i] = sqrt(sum)
      else
        $result[j,i] = sum / $result[i,i]
      end
    end
  end
}
```
PathWise Modeling Studio

- Encapsulate re-usable logic in UDFs and UDF libraries
- Define model outputs (e.g. Greeks)
PathWise Modeling Studio

- Define model outputs (e.g. Greeks)
PathWise Modeling Studio

- Commit model to SVN source code repo
PathWise Modeling Studio

- **Compile and deploy** model to GPUs
PathWise Modeling Studio

- Add GPU grid workers from the **Cloud**
PathWise Modeling Studio

- Generate sample Python script

```python
import global_config as cfg
import etl
import numpy as np

# Import model
import SMB_model_t_float as model

# Load inputs
DATA_DIR = 'C:\Users\smohammad\Documents\pms\GHM\'
model_data_nps = etl.load(DATA_DIR + 'data/model_data.nps', model.PRECISION)
market_data_nps = etl.load(DATA_DIR + 'data/market_data.nps', model.PRECISION)
iforce = model_data_nps['infores']

# Compute
session = model.OMG_session_t(cfg.COORDINATOR_ENDPOINT)
session.begin_session(model_data_nps, market_data_nps, modelType = model.ModelType.Default)
fmvs = session.compute_option_value(iforce)
session.end_session()

# Results
etl.save('output.nps', fmvs = fmvs)
print 'DONE'
```
- Run Python scripts from **PathWise Analytics Studio** (customized Python IDE)
PathWise Analytics Studio

- Run Python scripts from **PathWise Analytics Studio** (customized Python IDE)
PathWise Platform
PathWise Operations Control Center

Liability Performance Attribution Report 1: Liability FMV P&L

Info

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<tr>
<td>Liability Fair Market Value (CAD)</td>
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Total Liability FMV P&L

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<td>Total Liability FMV P&amp;L</td>
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Equity Risk Liability P&L

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<td>1.01</td>
<td>-0.52</td>
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<tr>
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<td>196,330,423.6</td>
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<td>427,245,893.4</td>
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PathWise Seriatim Real-Time Risk System
Conclusions
PathWise Modeling Studio Roadmap

- **Growing list of different Model Types**
  - Monte Carlo Models (✔ available now)
  - Data Parallel Models (✔ available now)
  - Nested Stochastic Simulation Models (✔ in progress)
  - Finite Difference Method PDE solvers
  - Support for non-trivial interactions between CUDA threads and between GPU grid nodes

- **A large set of GPU Model Libraries**
  - Economic Scenario Generators Library (✔ available now)
  - Standard and Exotic Hedge Program Instrument Library (✔ available now)
  - Hedge Strategy Library (✔ available now)
  - International Capital and Reserve Library (✔ in progress)

- **Automatic Model Differentiation (Adjoint Method) (✔ in progress)**
  - PWMS compiler can support automatic differentiation
  - Allows computation of model sensitiveness (Greeks) without re-simulation
Conclusions

- **High Productivity Computing (HPC)**
  - End-user focused tools for computational scientists
  - User-friendly, high productivity environments
  - Easy access to high throughput computing infrastructure (e.g. GPU cloud)

- **HPC requires**
  - Domain Specific Languages, Middleware, User-Interfaces
  - Even entire Domain Specific *Platforms*
  - Domain knowledge experts to design and support such tools
  - Investment in software tools

- **HPC provides**
  - Unprecedented user productivity and computational power