Potential Future Exposure and Collateral Modelling of the Trading Book Using NVIDIA GPUs

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Contents

• PFE Modelling explained
• Cash vs. No-Cash Collateral – The impact of haircut on the valuation
• GPU implementation
• Q & A
Consider the problem of calculating the potential **Future Collateral Requirements** of a large derivative book for a global asset manager

- **Purpose:** Provide a probabilistic approach of what may be the collateral requirements, on various granularity levels (fund level, counterparty level, division level, company level, etc.) and different time horizons, of clients portfolio

<table>
<thead>
<tr>
<th>Worse Case Scenario</th>
<th>Time Horizon</th>
<th>Collateral Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fund level</td>
<td>Division level</td>
</tr>
<tr>
<td>90%</td>
<td>1w</td>
<td>...</td>
</tr>
<tr>
<td>70%</td>
<td>1w</td>
<td>...</td>
</tr>
<tr>
<td>Average case</td>
<td>1w</td>
<td>...</td>
</tr>
</tbody>
</table>

- Additional information could be extracted as well. Collateral decomposition (Cash Collateral, Gov. Bonds, HY bonds, etc.), counterparty exposure, etc.
- Typically the portfolio will include interest rate swaps, swaptions, inflation swaps, cross currency swaps, equity options, CDSs, etc.
• **Potential Future Exposure (PFE)** is defined as the **maximum** expected credit exposure over a specified period of time calculated at some level of confidence. PFE is a measure of counterparty credit risk.

• **Expected Exposure (EE)** is defined as the **average** exposure on a future date

• **Credit Valuation Adjustment (CVA)** is an adjustment to the price of a derivative to take into account counterparty credit risk.

• **Collateral** can be considered any type of valuable liquid property that is pledged by the recipient as security against credit risk.
What do we need to build a ‘standard market’ PFE Model?

- A consistent simulation framework to project forward in time the interest rate curves. This will typically have **100-120 time-steps** and around **100k - 1mln scenarios**

- **Multiple stochastic processes** to simulate the FX exposure for every different currency in the portfolio, aligned with the time steps and the number of simulations of the interest rate scenarios. The same idea applies for the inflation rates but with less time steps

- **Correlate** all the Brownian motions calibrating on the market data and use the Cholesky decomposition to form multivariate normal random variables and project forward in time on every scenario

- **Re-evaluate** every position, on every time step, for **every scenario** and aggregate per netting set to calculate the Expected Exposure - EE

- Calculate the collateral changes based on the CSA agreement with every counterparty and available collateral on our pool
Example – Yield Curve Evolution
Example – Portfolio Value / Collateral Exposure over time

Total MTM Portfolio Value for All Scenarios

Collateral Exposure for All Scenarios

Simulation Dates
Example – Cash Collateral Exposure Distribution
95% & 80% Confidence level

Portfolio Collateral Exposure Profiles

- Max Exp
- Collateral Exp (95%)
- Collateral Exp (80%)
- Exp Exposure (EE)
Dimensionality of the problem

The need for GPU acceleration

- For every interest rate curve we will typically have **120+ simulation steps** (50 year horizon)
- To construct the **full term structure** on every simulation step we need **20-25 tenor points**
- We simulate **different forward and discount curves** in order to capture the stochastic credit spreads between the two curves (e.g. Sonia and Libor)
- The same applies for every currency in our portfolio along with a stochastic process to simulate the FX exposure
- Depending the degree of convergence we want to achieve we simulate **100k-1mln scenarios**

So for the simulation the dimensionality of the problem is:

$$n_{Sim} \times n_{Steps} \times n_{Tenors} \times n_{Curves} \times n_{Currencies}$$

$$1mln \times 120 \times 25 \times 15$$

For valuation purposes we have to take under consideration on top of that the size of the portfolio as well as the NO-Cash collateral
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CSA-dependent Valuation

• Trading with broker/fund-specific CSA
  – Credit-risky collateral requires a haircut to be applied on the notional amount (10%-30% depending on the credit quality, the broker and the fund)

  – For Bond Collateral we have to re-finance our position frequently, on every coupon payment
    • Additional management cost and liquidity issues
    • Repo markets to generate cash involves extra operations and costs and an agreed legal framework known as GMRA (global master repurchase agreement)

  – For more complex collateral products (CDSs / MBSs / Equities) the daily valuation is more complicated and more difficult to monitor

  – Foreign currency collateral implies cross-currency bootstrapping, incorporating cross-currency basis
Introducing the collateral pool on a multi-curve framework

– We assume that the fund on the previous example has a collateral pool with
  • £15mln Cash
  • £10mln of Gilts with 10% haircut
  • £10mln of AAA Bonds with 20% haircut
  • £20mln of HY Bonds with 30% haircut

– We now model a dynamic collateral portfolio that changes over time

– We re-evaluate the collateral on every time step, assuming there is no credit migration on the collateral posted

– Reconstruct the new collateral portfolio and simulate forward (additional computational complexity)
Bond Collateral Exposure Profiles - Results

Bond Collateral Exposure Profiles

Simulation Dates
Exposure (£mln)

Max Bond Collateral Exp (95%)
Bond Collateral Exp (95%)
Max Cash Collateral Exp (95%)
Cash Collateral Exp (95%)
Bond Collateral Exp (80%)
Cash Collateral Exp (80%)
Exp Exposure (EE)
When we include No-Cash collateral instruments in our collateral pool the complexity of the problem increases exponentially.

We now have a dynamic portfolio that changes continuously, adding an optimization depending on the collateral decision on every step we re-evaluate the portfolio.

The moment that the first No-Cash Collateral is posted either from us or our counterparty, we need to start simulating the credit migration risk associated with that bond.

*This is clearly an computational intensive exercise, ideal for parallelization and acceleration using GPUs.*
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NVIDIA GPU Computing: Graphical Processing Units
Decomposing the problem

• Preprocessing the portfolio
  – Fetch all the interest rate curve / FX data live from Bloomberg / Thomson Reuters
  – Read the portfolio and construct all the asset classes and projected cash-flows
  – Preprocess the portfolio for different calendars, payment/receive/reset frequencies

Mainly a task we use only CPUs for portfolios with less than 10000 derivatives. Banks with larger portfolios may consider parallelize this step as well.

• Monte Carlo simulation
  – Perform the simulation for the various stochastic processes and construct the full term structure for every simulation step

• Post process the portfolio
  – Re-Evaluate the portfolio on every scenario and simulation step adjusting for the collateral posted from either us or our counterparties

Acceleration using GPUs is essential!
GPU Implementation

Four GPU kernels (parallelized dimensions)
- Interest rate path generation (1mln paths)
- Rates conversion (number of paths x number of tenors)
- Zero rates (number of paths x cash flow dates)
- Forward rates (number of paths x cash flow dates)

Number of paths per thread block: 1024 / number of tenors
Number of thread blocks per thread grid: paths x tenors / 1024

<table>
<thead>
<tr>
<th>T_1</th>
<th>T_2</th>
<th>T_3</th>
<th>...</th>
<th>T_{N-2}</th>
<th>T_{N-1}</th>
<th>T_N</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_1(T_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>r_2(T_1)</td>
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<tr>
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<td>r_{k-1}(T_3)</td>
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<td>r_{k-1}(T_{N-1})</td>
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<tr>
<td>r_k(T_1)</td>
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<td>r_k(T_{N-2})</td>
<td>r_k(T_{N-1})</td>
<td>r_k(T_N)</td>
</tr>
</tbody>
</table>

Monte Carlo simulation evolution – N time steps
Parallel Architecture

Thread (0, 0)

Interest Rate(0, 0)
(Shared Memory)

Generation (Write)

Valuation (Read)

Thread (0, 1)

Interest Rate(0, 1)
(Shared Memory)

Generation (Write)

Valuation (Read)

Thread (path, step)

Interest Rate(path, step)
(Shared Memory)

Generation (Write)

Valuation (Read)

Tenor 0

Tenor N

Portfolio Valuation

Tenor 0

Tenor N

Portfolio Valuation

Tenor 0

Tenor N

Portfolio Valuation
Acceleration Results

- CPU execution times & Speedup
- Test Environment:
  - CPU: an Intel Xeon Sandybridge X5650 2.66 Ghz (16 cores)
  - an NVIDIA Tesla K40

<table>
<thead>
<tr>
<th>Number of paths</th>
<th>Total CPU time (seconds)</th>
<th>Total GPU time (seconds)</th>
<th>Speed-up (Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>5.1</td>
<td>22.8</td>
<td>-</td>
</tr>
<tr>
<td>20000</td>
<td>8.8</td>
<td>19.6</td>
<td>-</td>
</tr>
<tr>
<td>30000</td>
<td>13.1</td>
<td>21.4</td>
<td>-</td>
</tr>
<tr>
<td>40000</td>
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<td>-</td>
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<tr>
<td>50000</td>
<td>21.5</td>
<td>20.2</td>
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<tr>
<td>100000</td>
<td>43.3</td>
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<td>2x</td>
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<td>500000</td>
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<tr>
<td>1000000</td>
<td>503.6</td>
<td>22.2</td>
<td>25x</td>
</tr>
</tbody>
</table>
Conclusions

• Multi-curve modelling is here to stay
  – Impact varies, but dual curve discounting is essential for accurate pricing

• No-Cash Collateral management is not trivial
  – **Multiple** broker/fund specific **CSA agreements**
  – **Haircuts** do have a significant impact on collateral and fund’s **liquidity**
  – The **dynamic** portfolio grows the computational complexity exponentially
  – Monte Carlo framework provides a sensible solution

• High Performance Computing using GPUs is essential
  – Significant increases the performance of Monte-Carlo simulation
    • Memory allocation and data transfer problems have to be taken under consideration
  – **Total cost of ownership**
  – Solution depends on the business case
  – In-house development has significant **Person Risk**
    • **Outsourcing?**
Thank you / Any questions?
References

Past performance is not a guide to future returns. The value of investments, and the income from them, can go down as well as up and your clients may get back less than the amount invested.

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