Machine Learning with GPUs:

Fast Support Vector Machines without the Coding Headaches

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NVIDIA GPU Technology Conference
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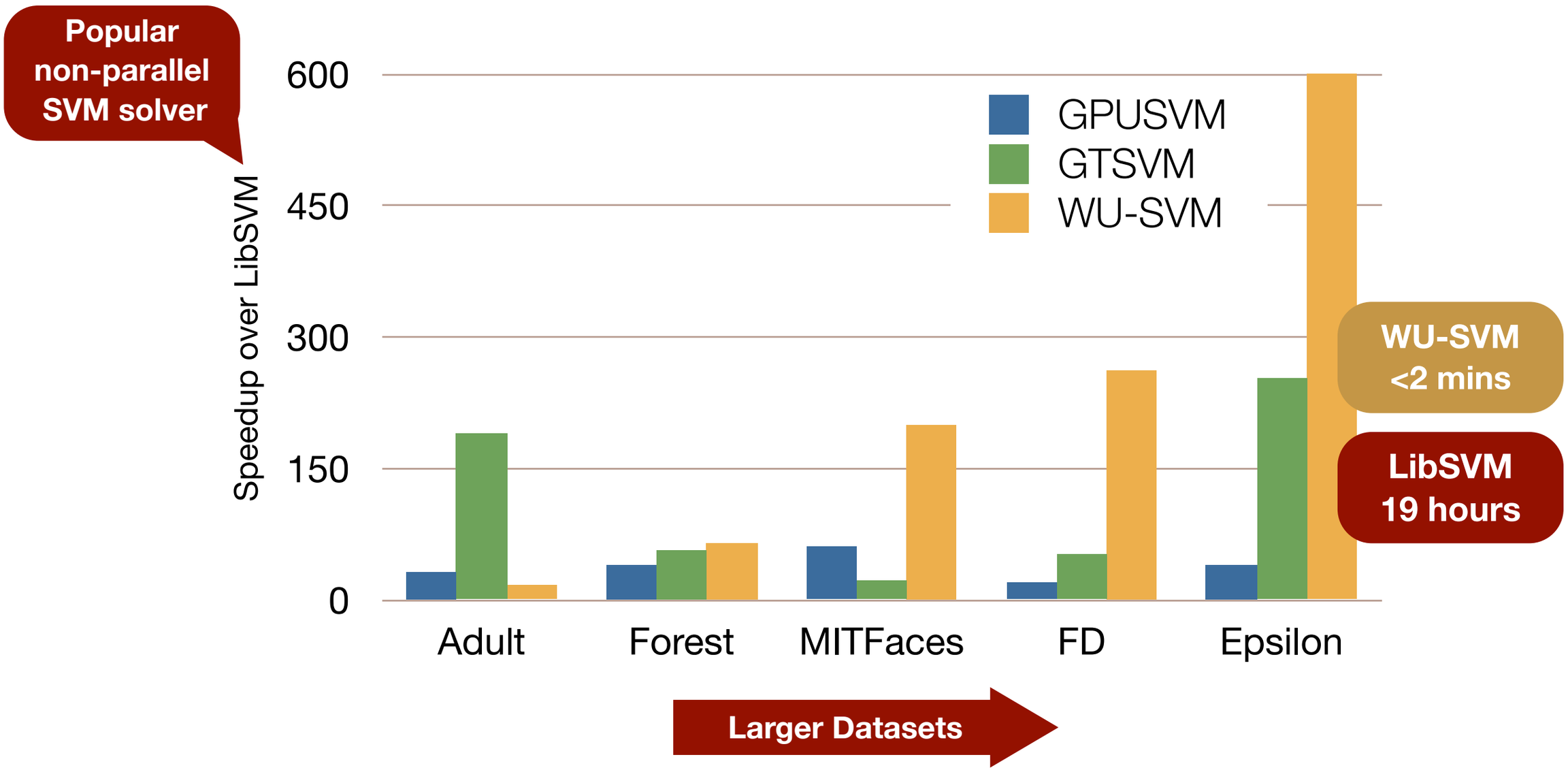
Starting with the last slide...



Takeaways

A bit of machine learning...

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An example of choosing a “high-efficiency” algorithm (plays well with great code like CUBLAS)...

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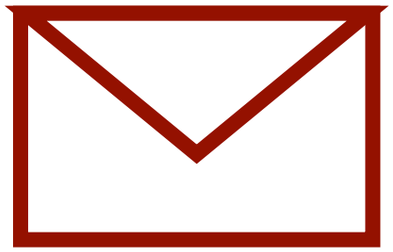
A bit of machine learning...

An example of choosing a “high-efficiency” algorithm (plays well with great code like CUBLAS)...

Some really fast SVM training code...

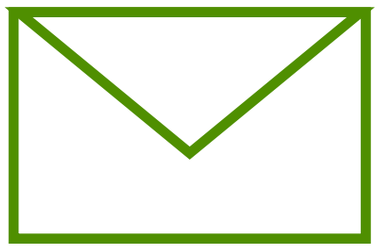
Machine Learned Classification

Learn to distinguish two types of things...



“Spam” emails

vs



“Ham” emails

Handwritten 3s

vs

Handwritten 4s

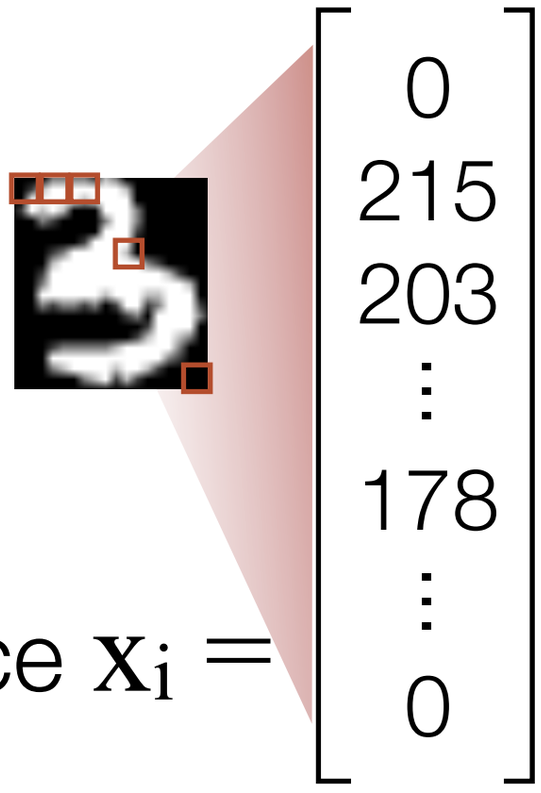
My face

vs

Other faces

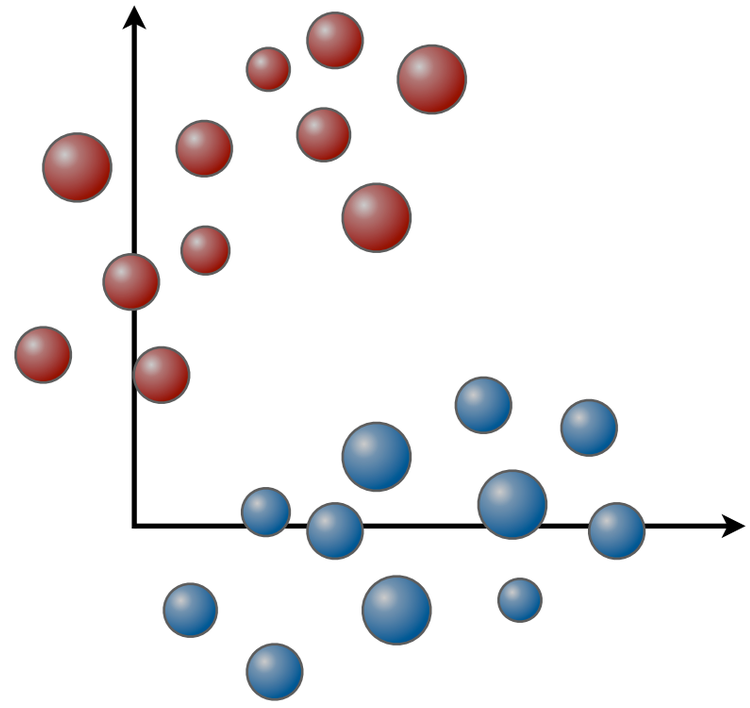
Machine Learned Classification

Vector feature representation



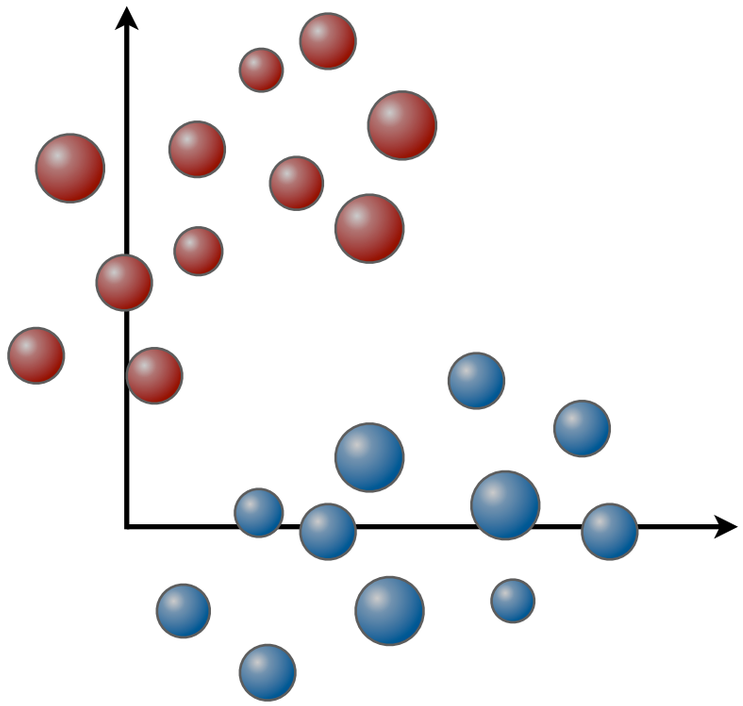
$$\text{Instance } \mathbf{x}_i = \begin{bmatrix} 0 \\ 215 \\ 203 \\ \vdots \\ 178 \\ \vdots \\ 0 \end{bmatrix}$$

$$\text{Label } \mathbf{y}_i = +1$$



Machine Learned Classification

Training data



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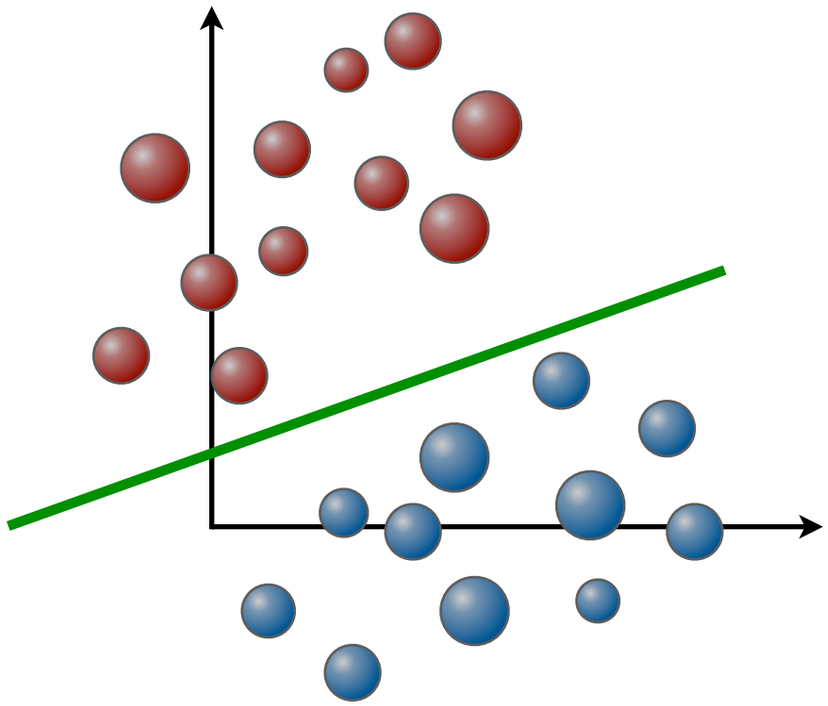

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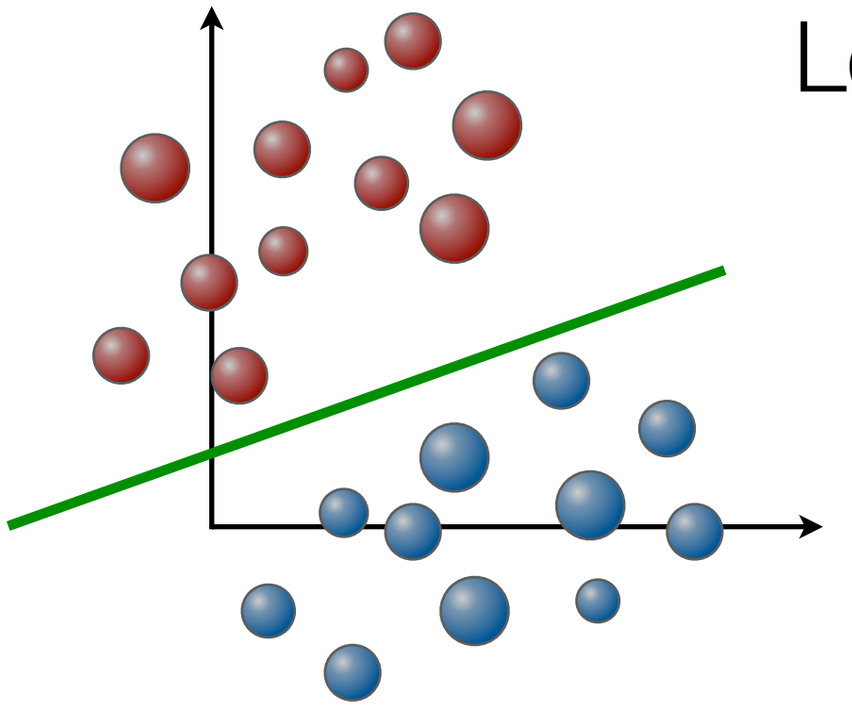
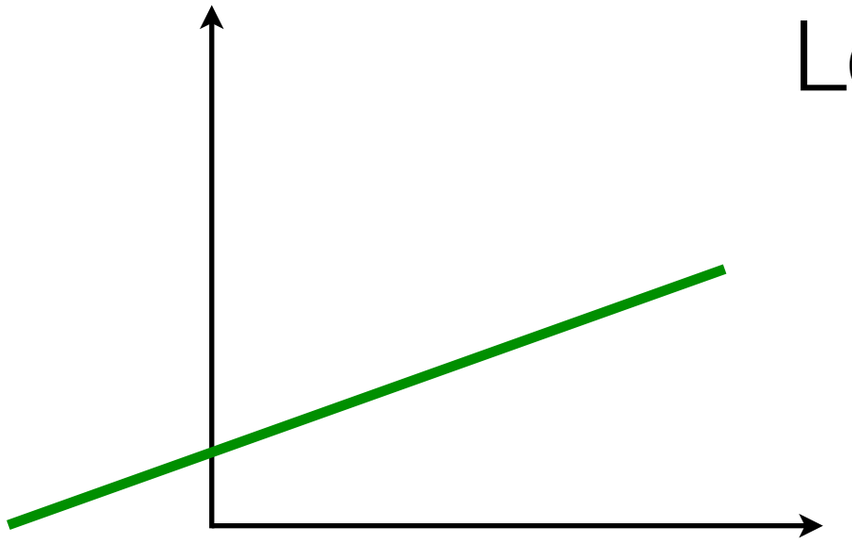


Learn model $(\mathbf{w},\mathbf{b})$

$$\mathbf{w}^\top \mathbf{x}+\mathbf{b}=0$$

Machine Learned Classification



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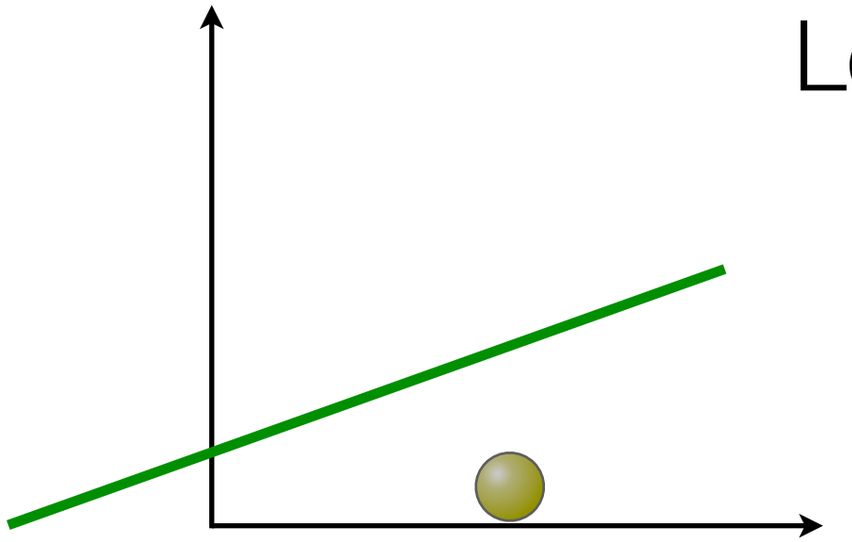

Learn model $(\mathbf{w},\mathbf{b})$

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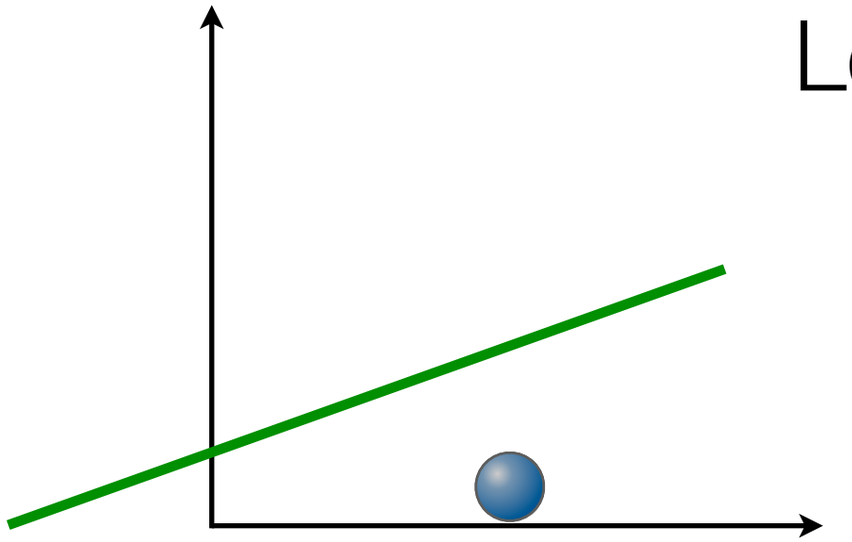
$$\text{sign}(w^\top x+b) > 0?$$

Machine Learned Classification



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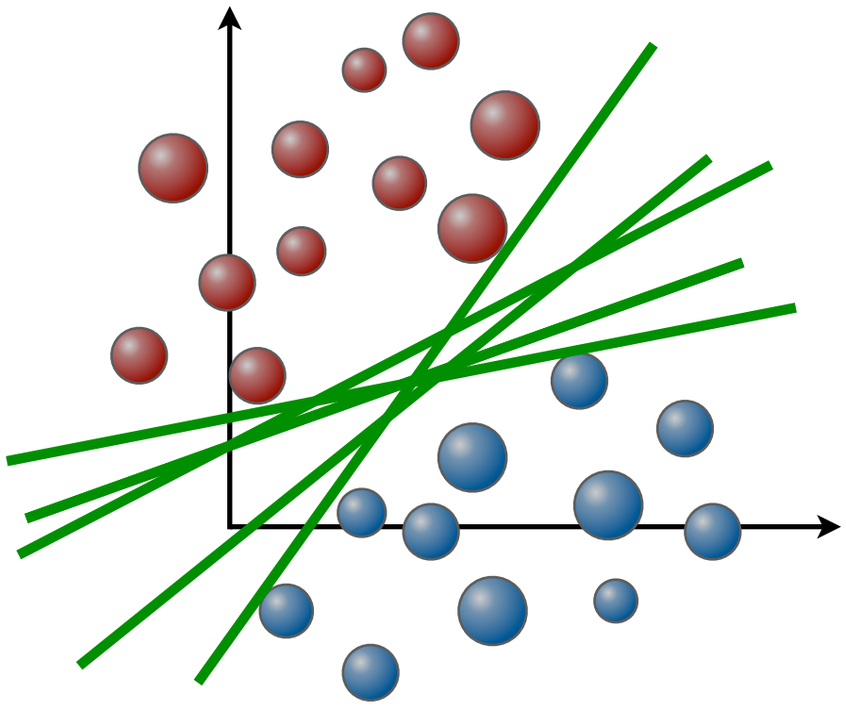
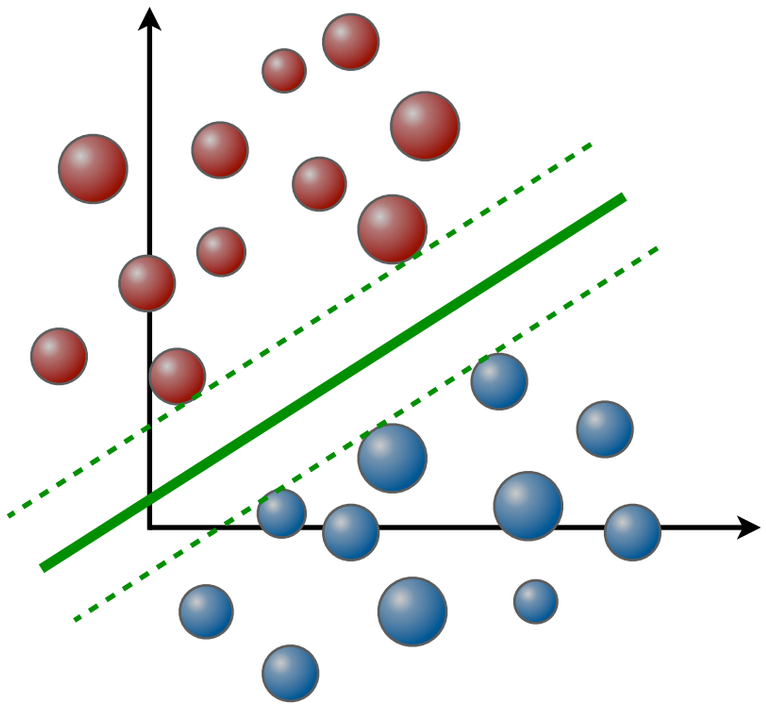
Machine Learned Classification

Which model?



Support Vector Machines [Cortez & Vapnik, 1995]

"Maximum margin"



"Margin" term

Error penalty

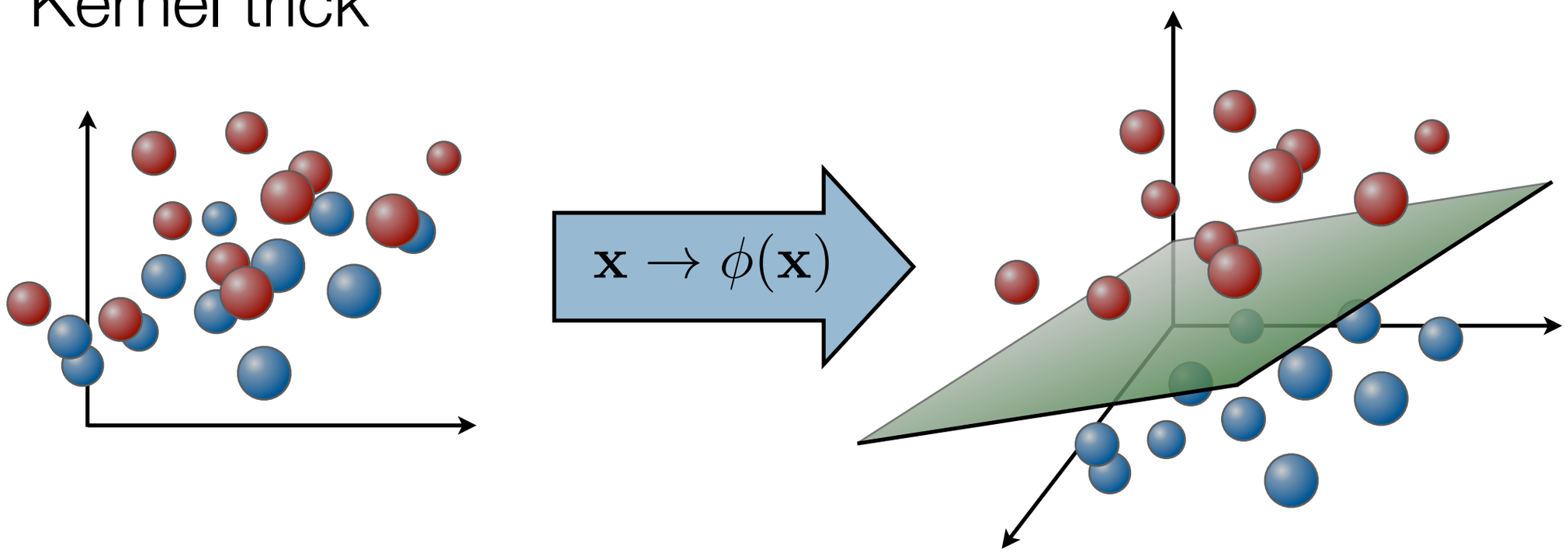
$$\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\mathbf{w}^\top \mathbf{x}_i + b))$$

True Label

Prediction

Kernel Support Vector Machines

"Kernel trick"



Non-linear feature projection + linear model =
non-linear decision boundary

Kernel Support Vector Machines

$$\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\mathbf{w}^\top \phi(\mathbf{x}_i) + b))$$

Kernel Support Vector Machines

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high/infinite dimensionality

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Dual

$$\max_{C \geq \alpha_i \geq 0} -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j \, \phi(\mathbf{x}_i)^\top \phi(\mathbf{x}_j) + \sum_{i=1}^{n} \alpha_i$$

$$s.t. \sum_{i=1}^{n} \alpha_i y_i = 0$$

Kernel Support Vector Machines

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$$k(\mathbf{x}_i, \mathbf{x}_j) = e^{-\gamma||\mathbf{x}_i - \mathbf{x}_j||^2}$$

e.g. RBF kernel

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Kernel function

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Quadratic program,
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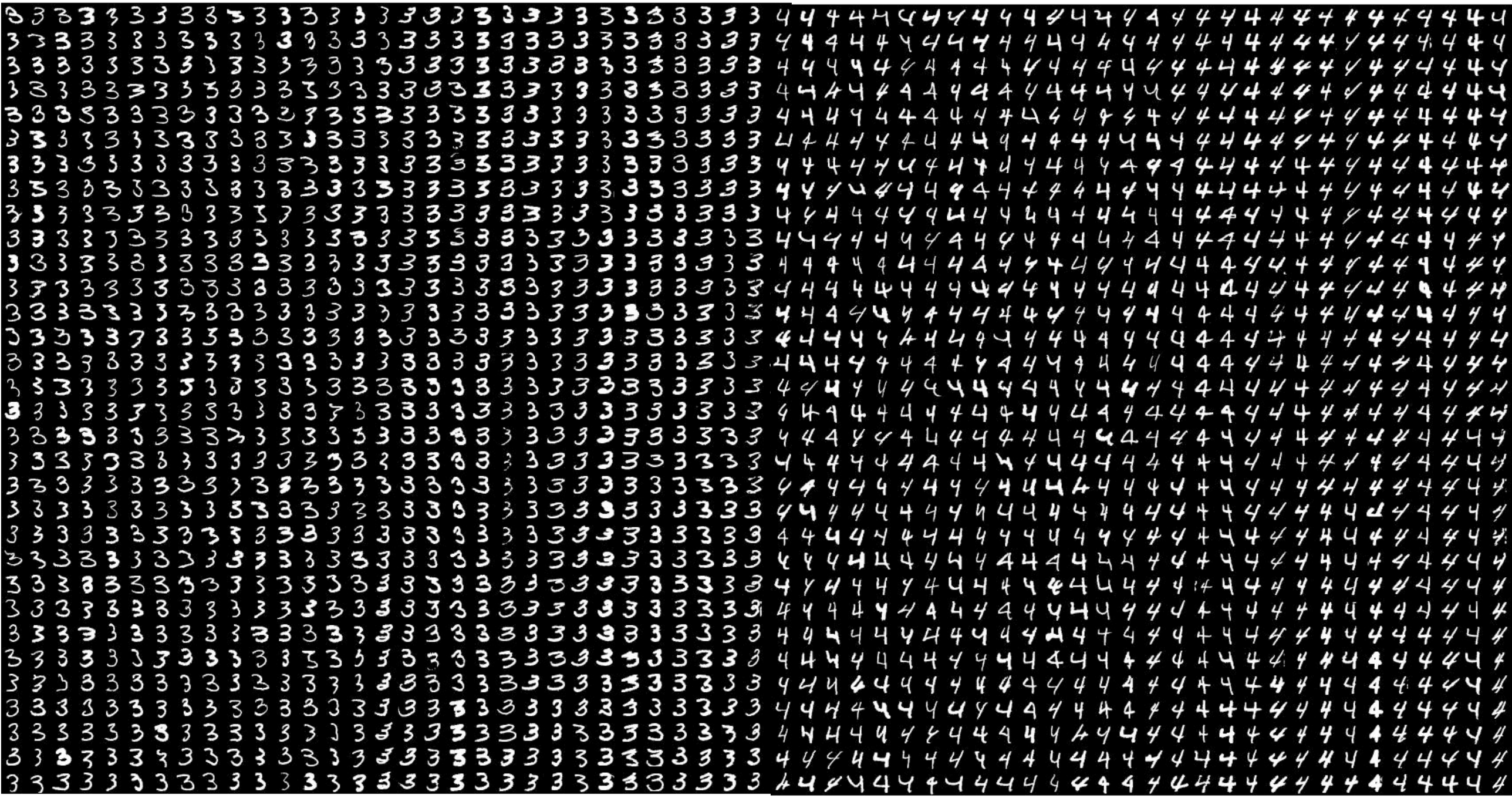
Many lightweight iterations

What about large training sets?



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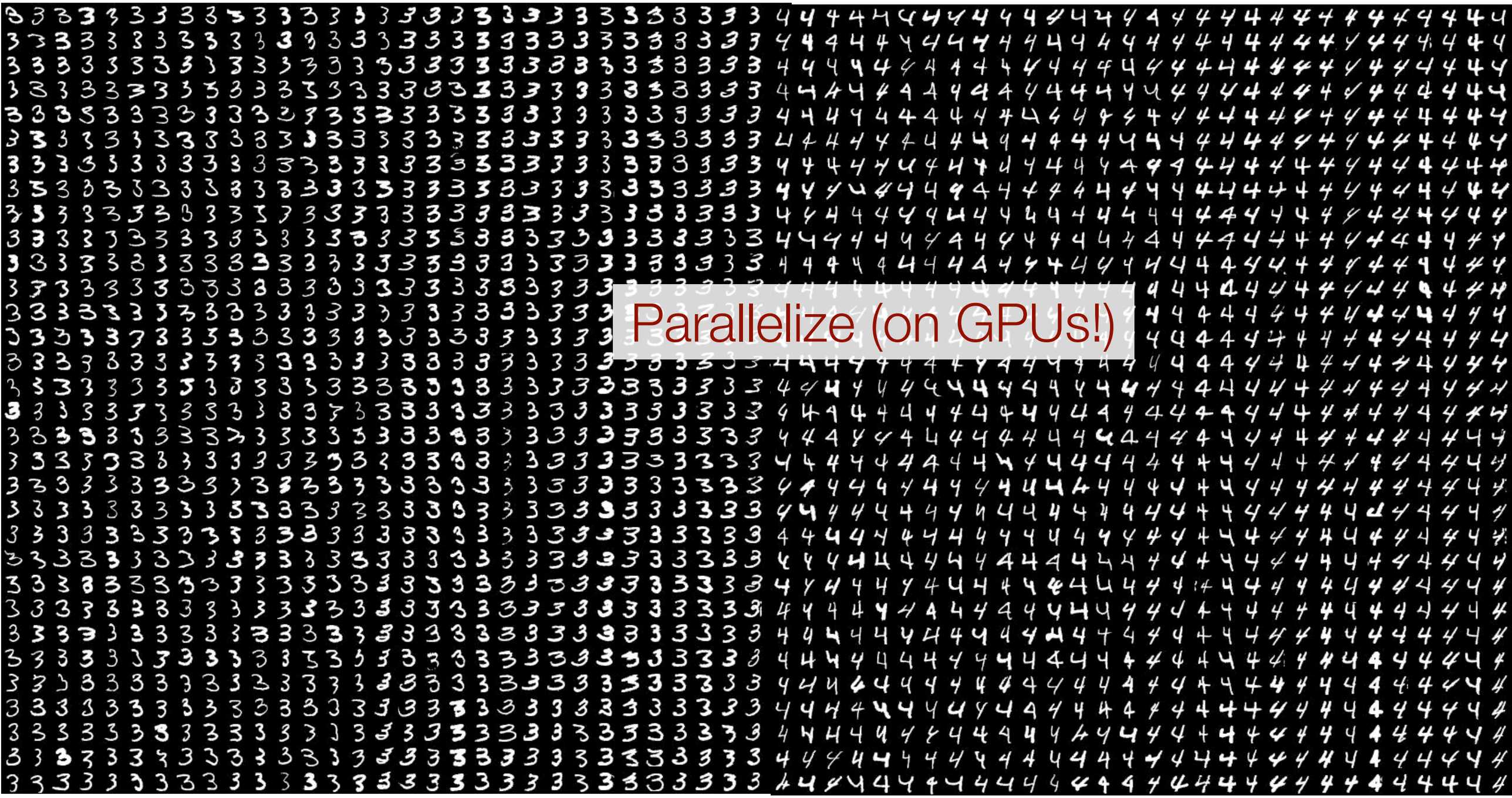
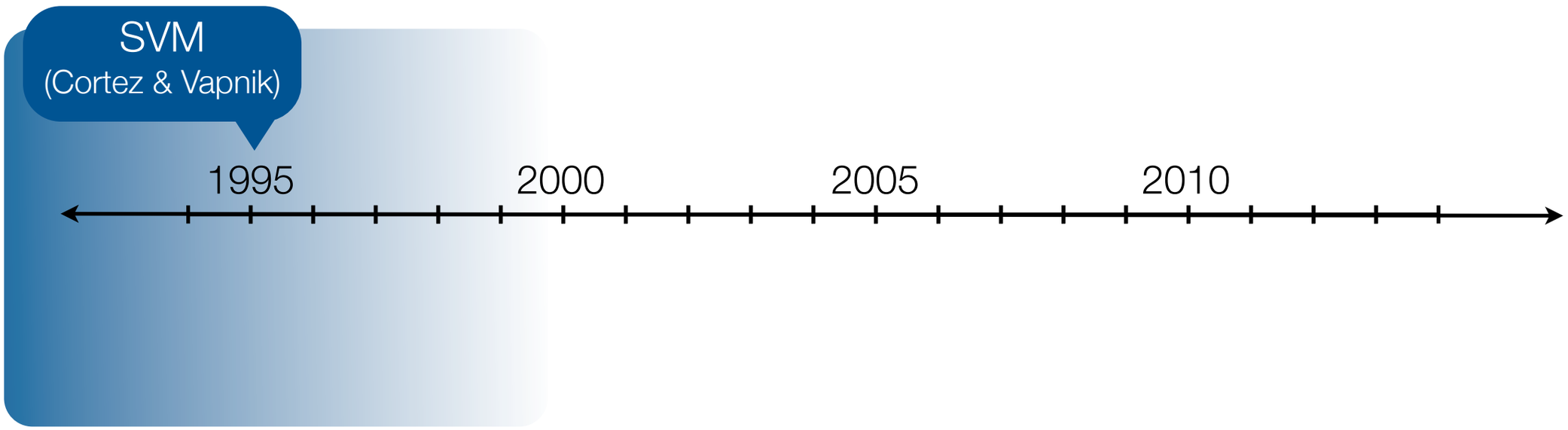


Can we do better?

- Are dual decomposition methods the best candidates for parallelization?
  - well-known and understood solvers
  - but, many lightweight iterations
  - complex hand-coded solutions



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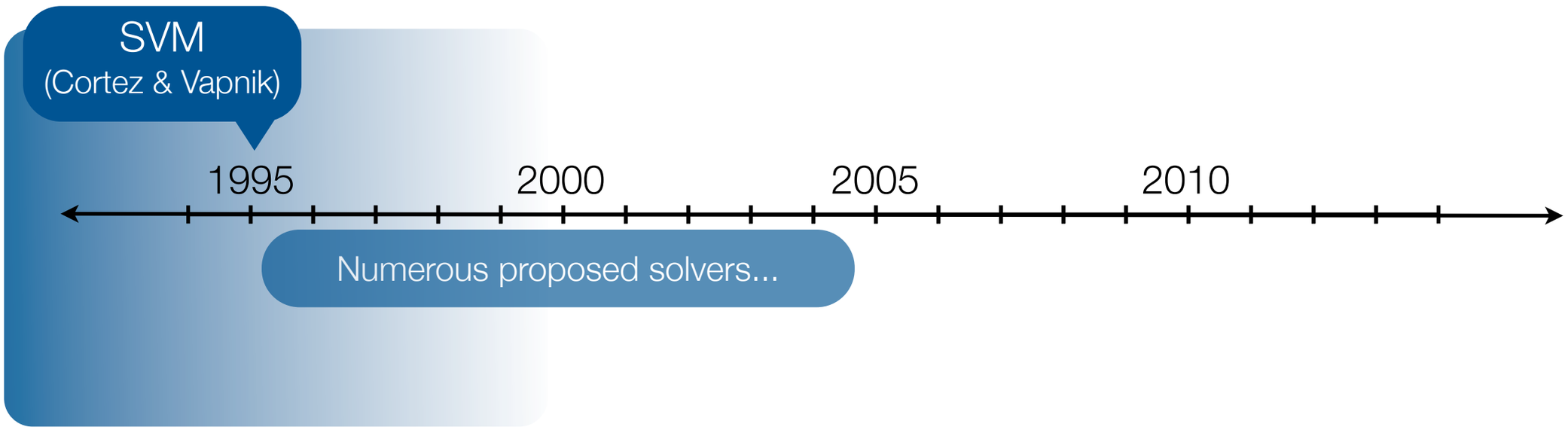

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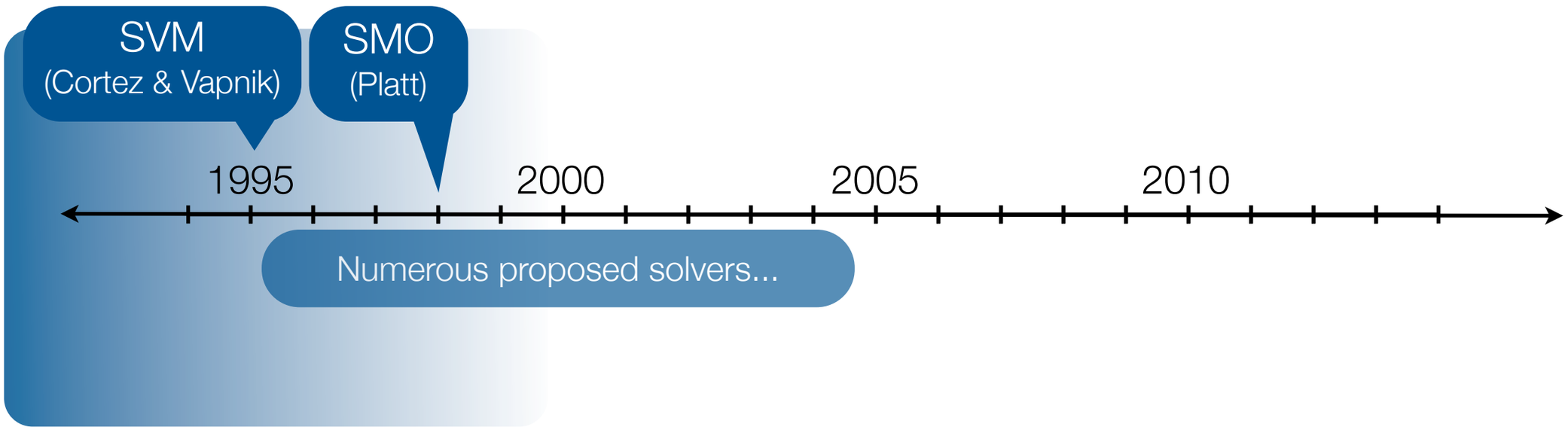

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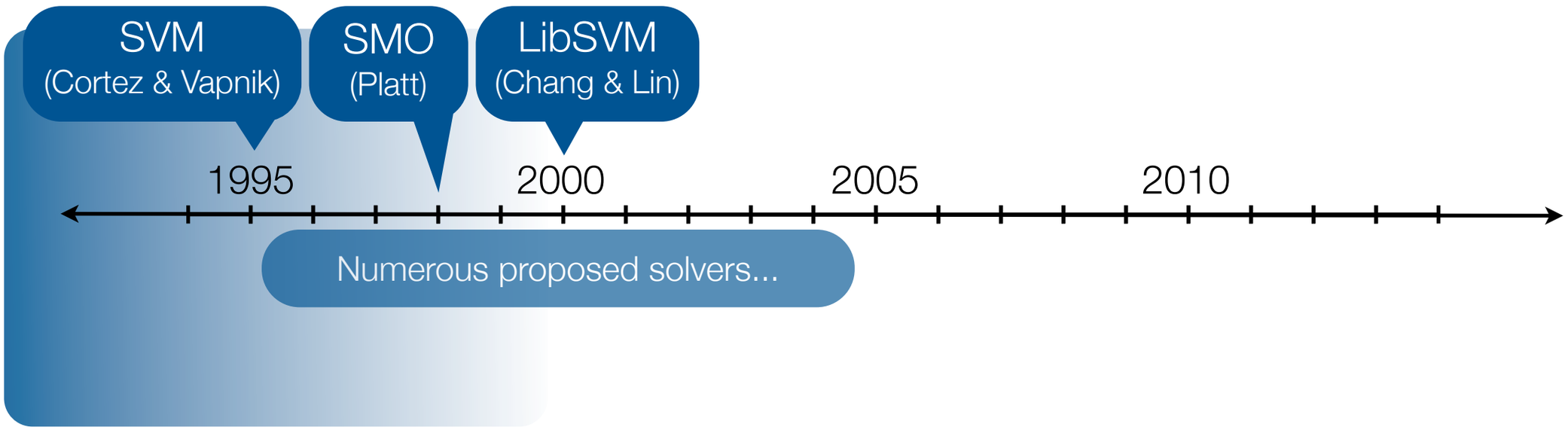

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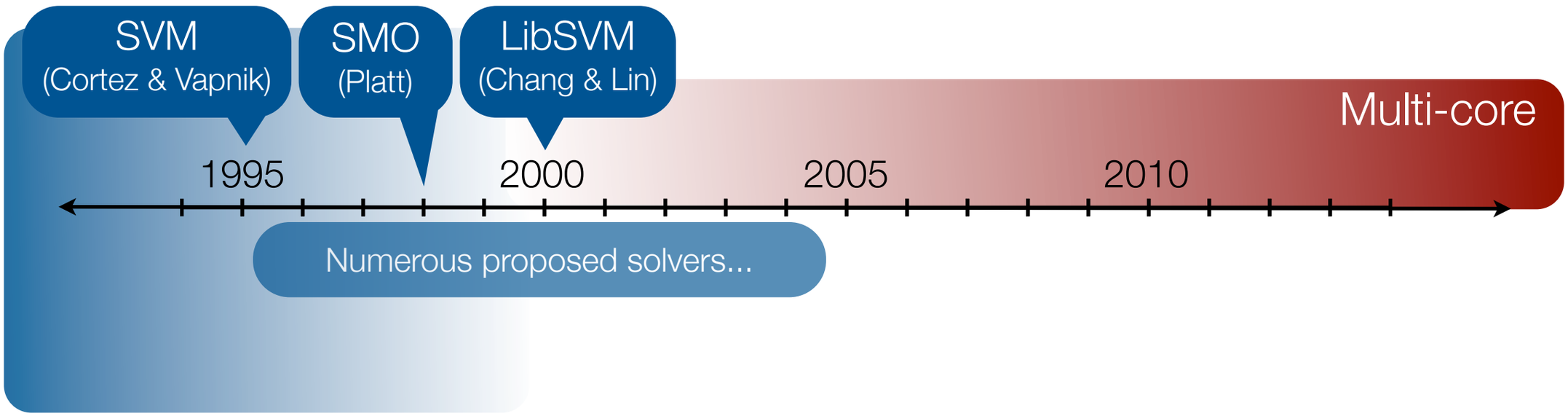

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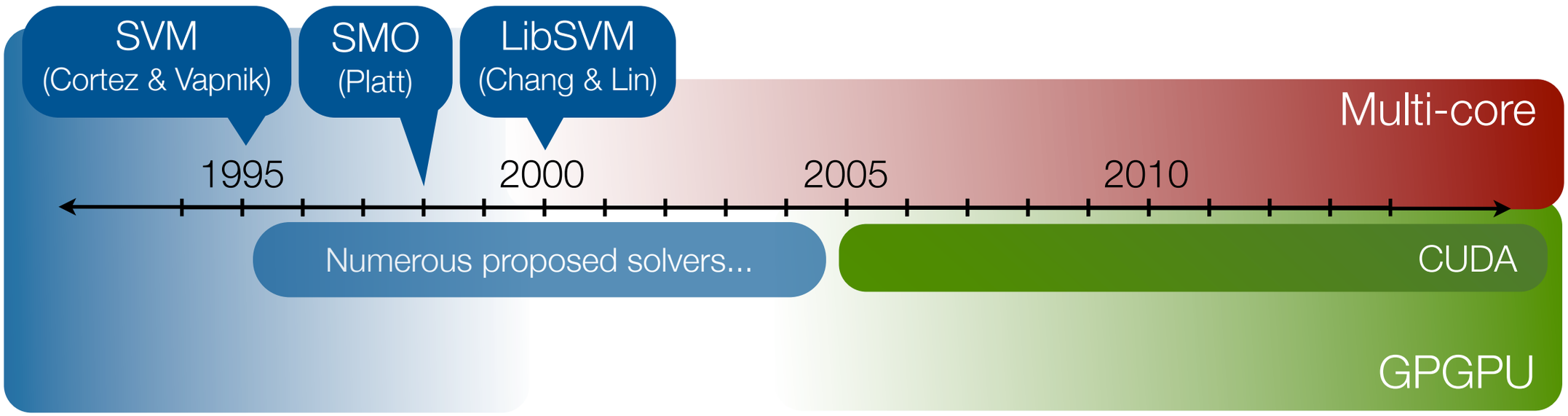

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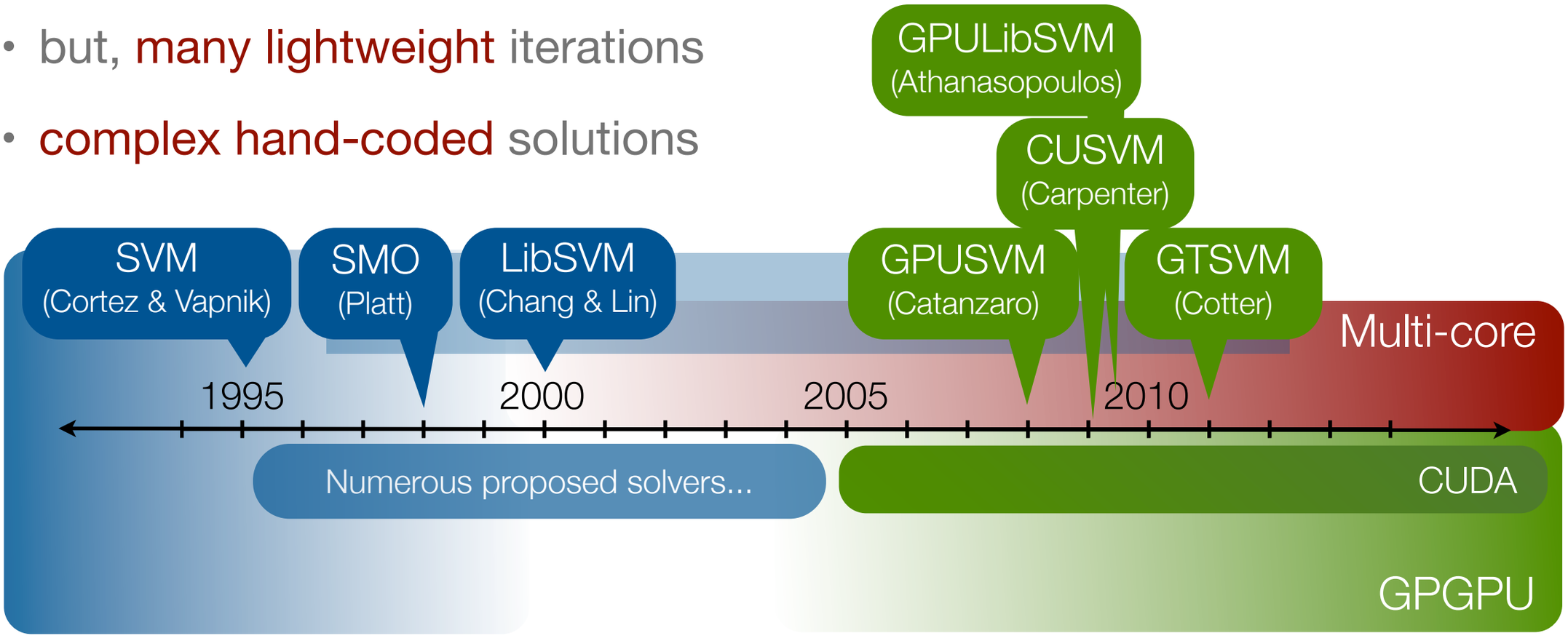

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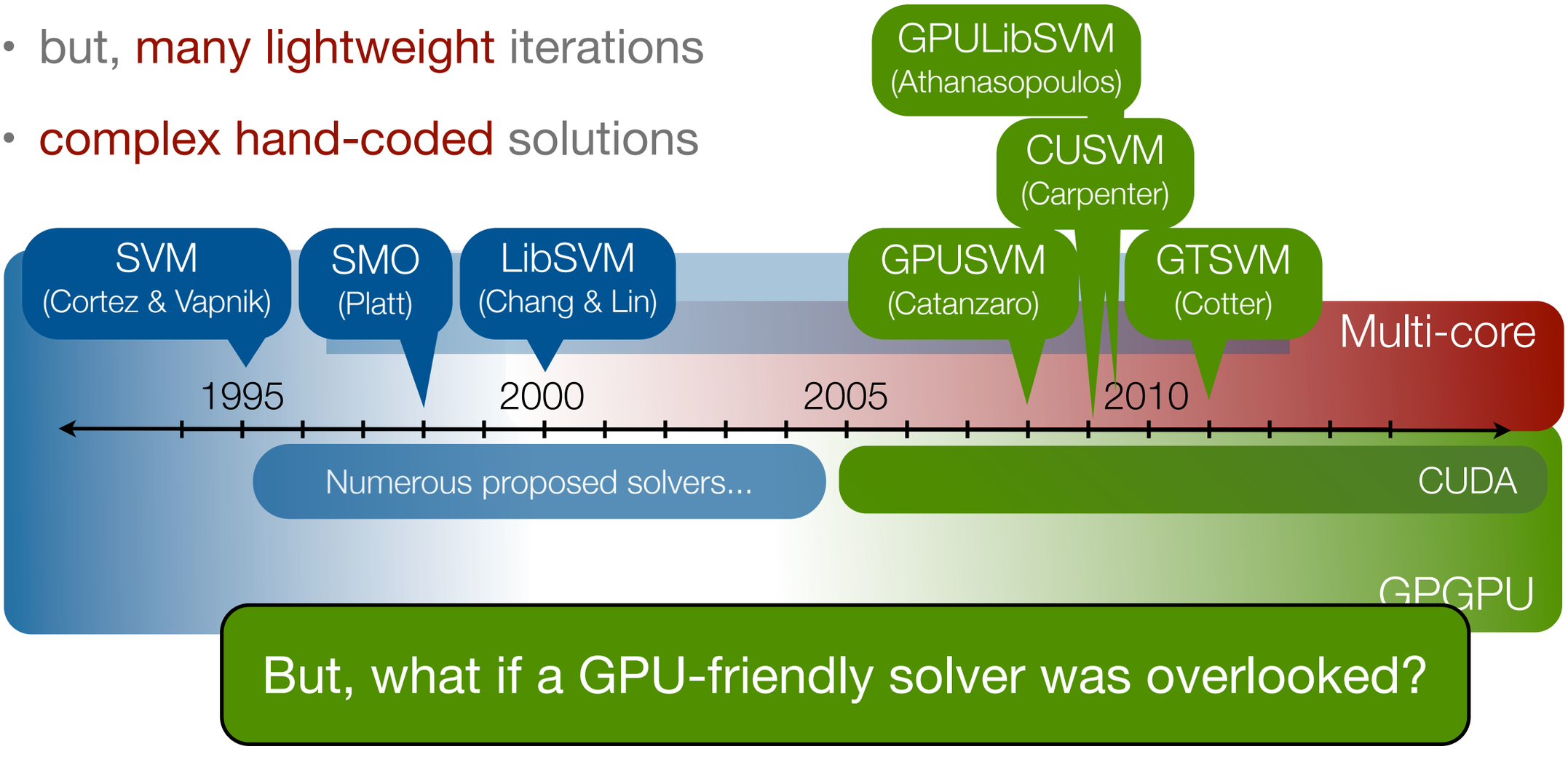

Can we do better?

- Literature search

  - ☐ few, dense iterations

  - ☐ easily parallelized core computations (e.g. linear algebra)

  - ☐ approximation (since traditional SVMs scale number of parameters with the number of training instances)

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Solves approximate SVM in primal form using the representer theorem [Chapelle, 2000]

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  ☑ approximation (since traditional SVMs scale number of parameters with the number of training instances)
  -- control approximation by number of kernel basis vectors

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$$\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n} \max(0, 1 - y_i(\mathbf{w}^\top \phi(\mathbf{x}_i) + b))$$

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Small basis set ("support vectors")

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SP-SVM Objective

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$$\min_{\boldsymbol{\beta},b} \frac{1}{2}\boldsymbol{\beta}^\top \mathbf{K}_{\mathcal{J}\mathcal{J}}\boldsymbol{\beta} + \frac{C}{2}\sum_{i=1}^{n} \max(0, 1 - y_i(\boldsymbol{\beta}^\top \mathbf{k}_{\mathcal{J}i} + b))^2$$

1xB

BxN

$$\mathbf{g} = \mathbf{K}_{\mathcal{J}\mathcal{J}}\boldsymbol{\beta} - C\mathbf{K}_{\mathcal{J}\mathcal{I}}(\mathbf{y}_{\mathcal{I}} - \boldsymbol{\beta}^\top \mathbf{K}_{\mathcal{J}\mathcal{I}})$$

Gradient

$$\mathbf{H} = \mathbf{K}_{\mathcal{J}\mathcal{J}} + C\mathbf{K}_{\mathcal{J}\mathcal{I}}\mathbf{K}_{\mathcal{J}\mathcal{I}}^\top$$

Hessian

Matrix/Vector Multiply

Matrix/Matrix Multiply

Linear System Solver

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Hessian

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$$\boldsymbol{\beta}' = \boldsymbol{\beta} - \mathbf{H}^{-1}\mathbf{g}$$

Newton Optimization

Matrix/Vector Multiply

Matrix/Matrix Multiply

Linear System Solver

Implementation

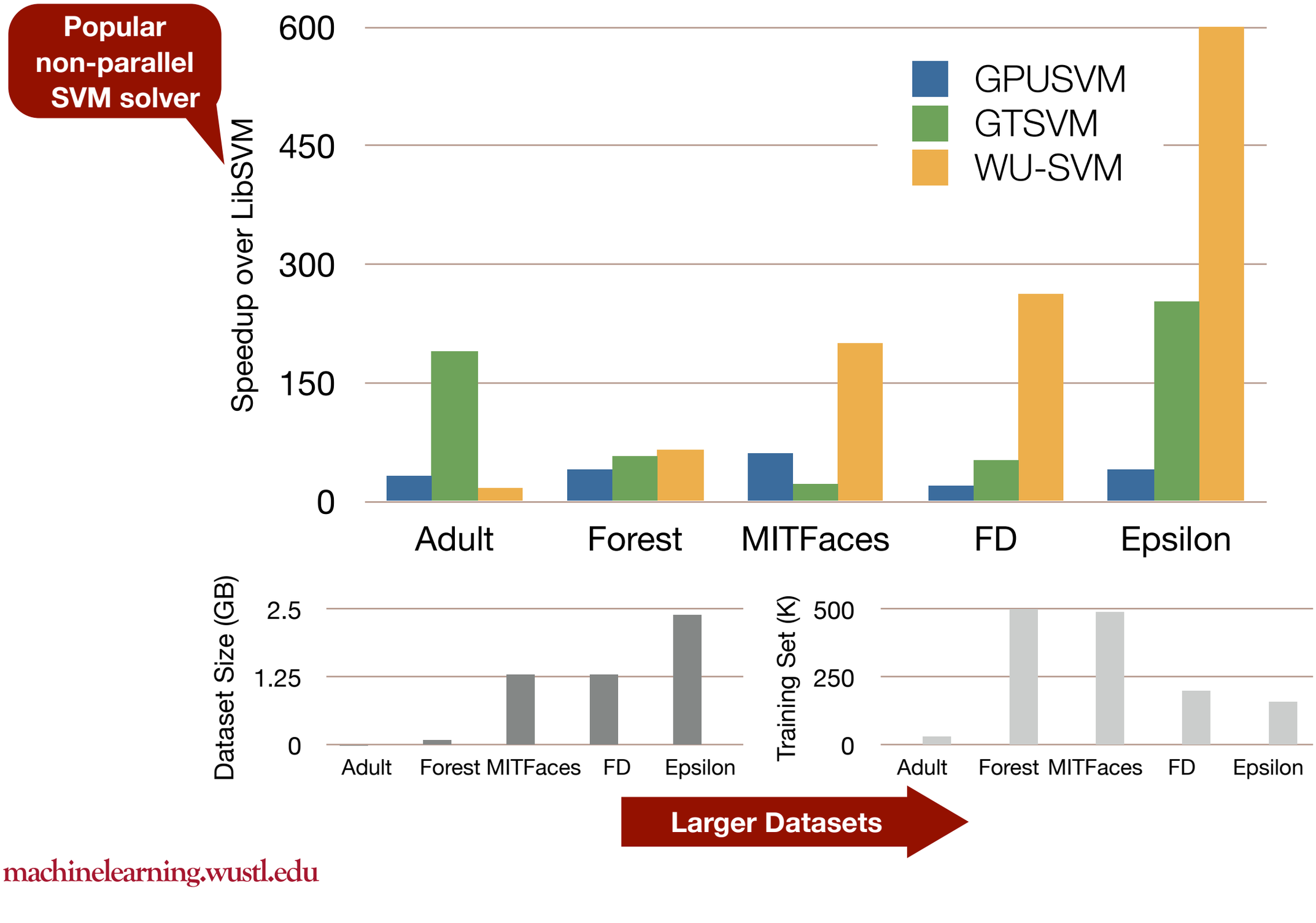
- CUBLAS!
  - No tedious hand-coding of kernels
  - Takes advantage of insider knowledge (CUBLAS by NVIDIA)
  - Automatically updates to latest graphics cards and BLAS library updates
  - (Almost) automatically transfers to other platforms (with BLAS implementations, i.e. multicore)

WU-SVM

- SP-SVM in C++/(CU)BLAS with open source BSD license
- Compatible with multicore BLAS implementations
- Implements LibSVM interface (command line, dataset format, output files)
- Written by undergraduate coders: Gabriel Hope, Nicholas Kolkin, and Jack Hessel

http://www.tinyurl.com/wu-svm

Experiments



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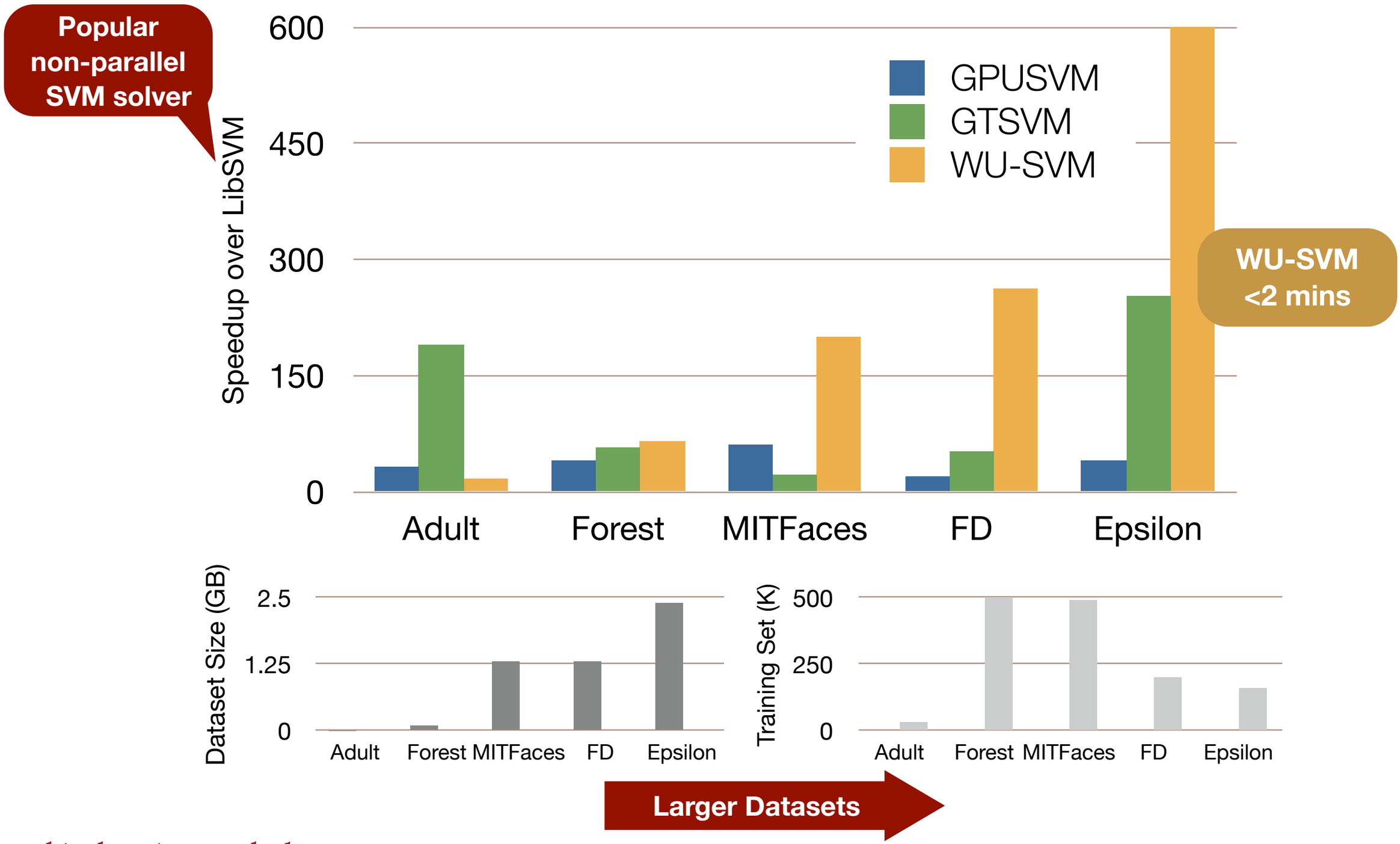

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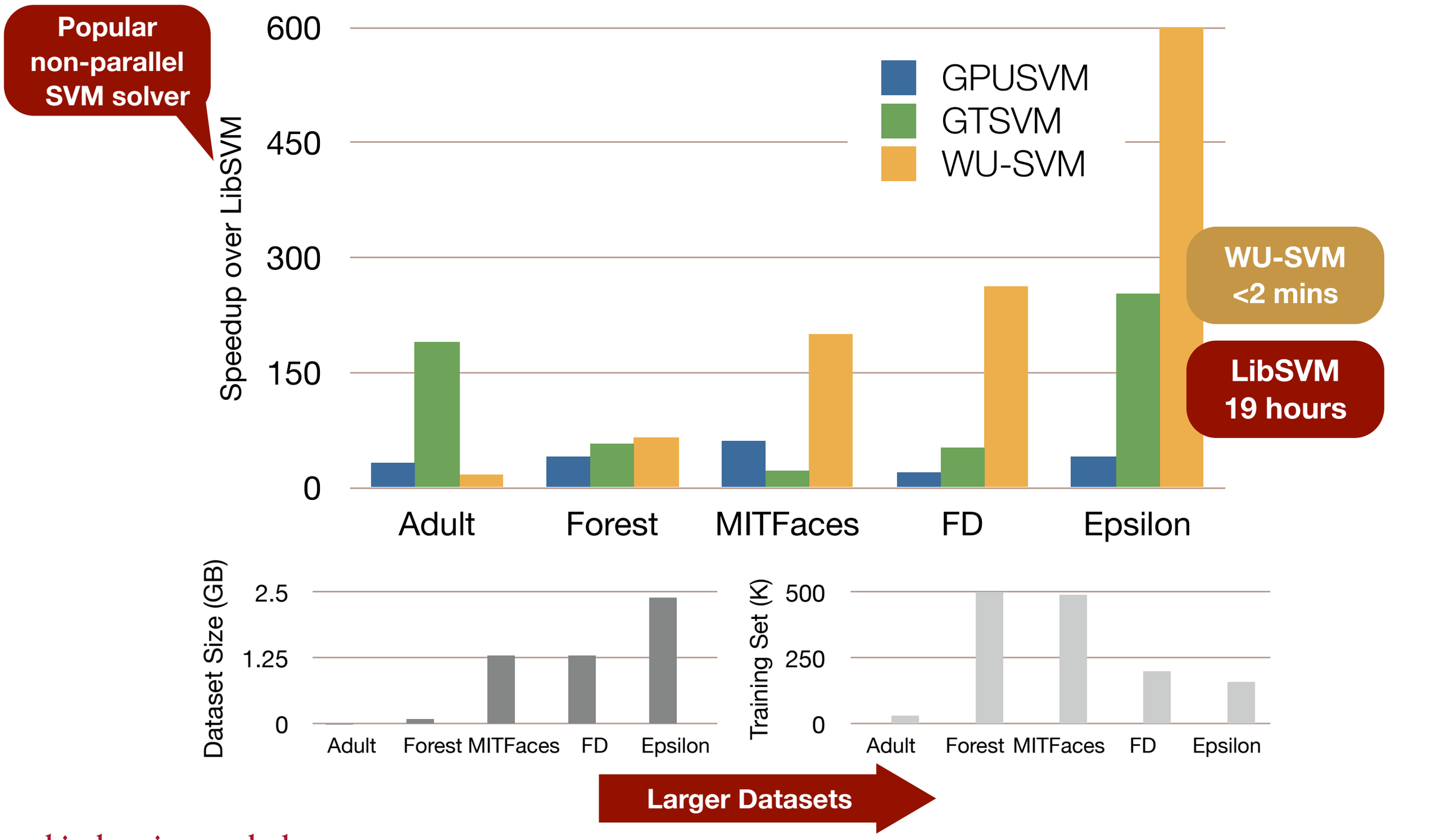

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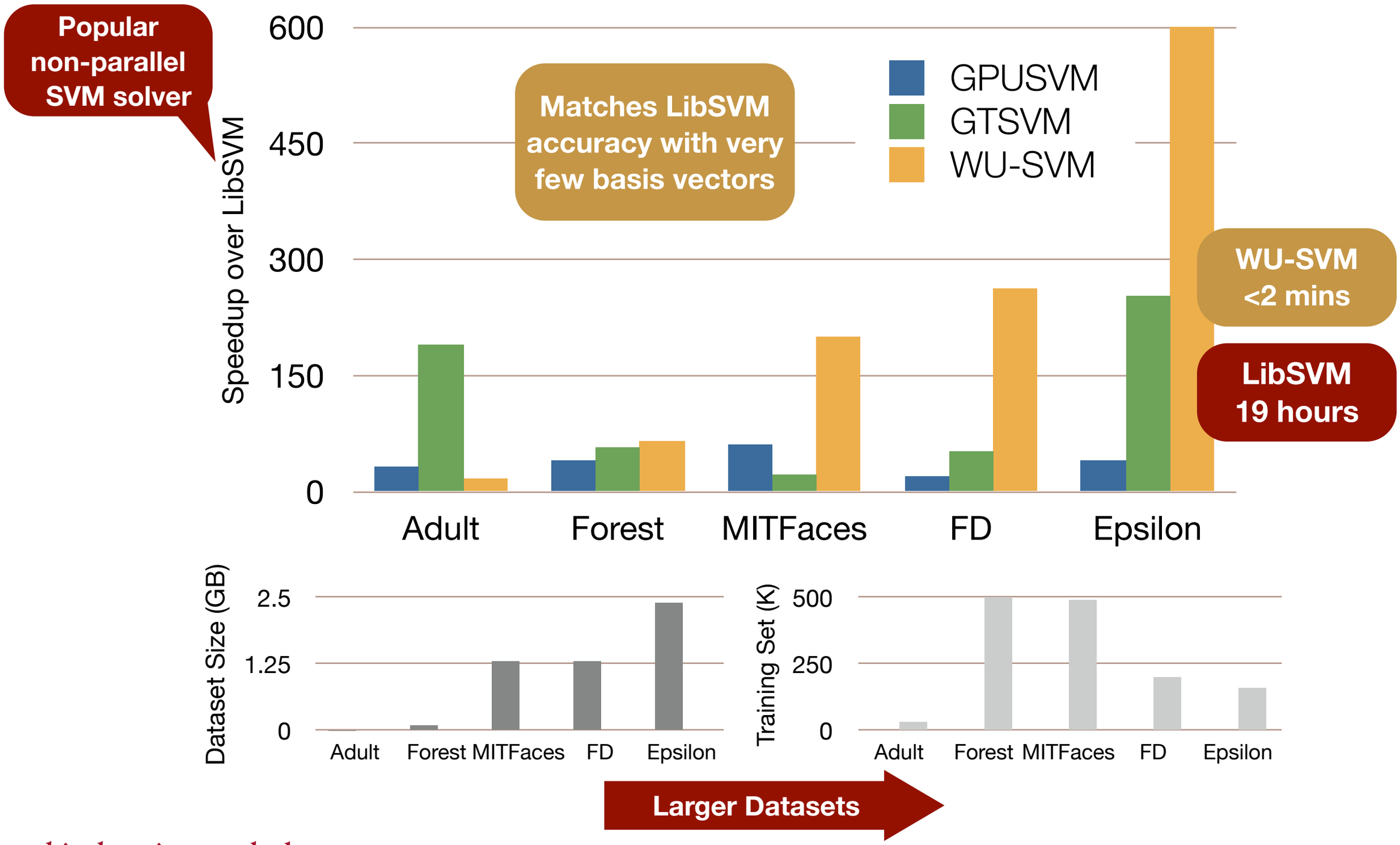

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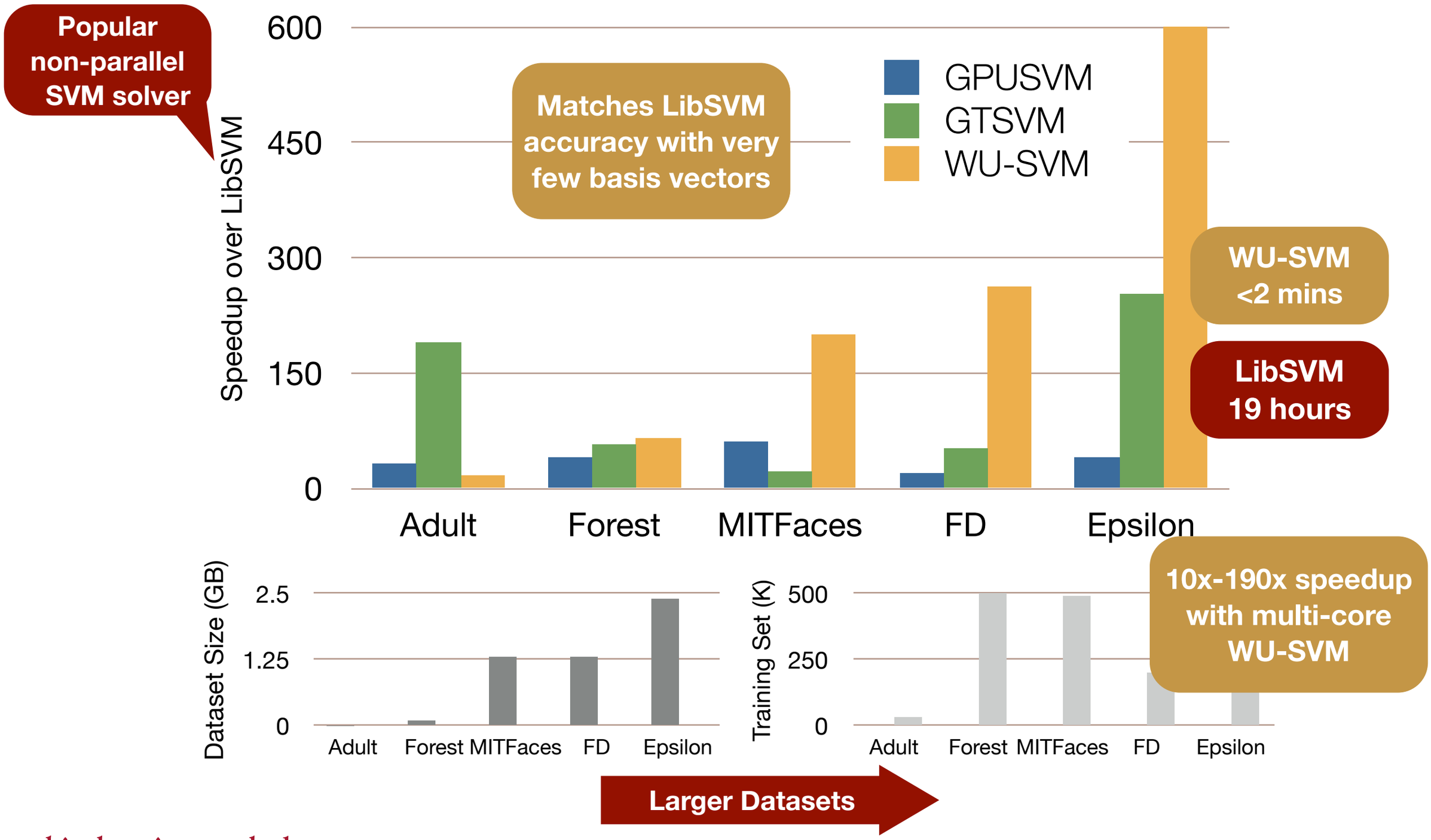

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  - Highly parallel since it's the right type of operation (largely dense linear algebra)
  - Very fast since well-designed/engineered code already exists
  - Easy to write/maintain/port to new platforms (since these libraries are standardized and constantly updated for new hardware)
- All we did was put the right pieces together (Keerthi's SP-SVM + CUBLAS), but it makes you wonder what other methods could benefit from a similar approach...

http://www.tinyurl.com/wu-svm

Thank you! Questions?

Thanks to Gabriel Hope, Nicholas Kolkin, Jack Hessel, Jacob Gardner, John Tran (NVIDIA), Kunal Agrawal, and Kilian Weinberger

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