



# Acceleration of $l_1$ -Regularized SPIRiT MRI Reconstruction by Fast DWT and GPU Computing

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## Abstract

We propose a novel GPU implementation of the Discrete Wavelet Transform (DWT) to accelerate compressive sensing-based MRI reconstruction. Routinely, the implementation of DWT involves two kinds of operations: pixel-dependent computation and image-size-dependent computation. In existing implementations of compressed sensing-based MRI reconstruction, DWT is typically implemented by general-purpose GPU libraries such as CUDA SDK, which performs both pixel-dependent computation and image-size-dependent computation at each execution. In this poster, we take advantage of the fact that the image size is fixed in MRI reconstruction, and thus we can perform the image-size-dependent computation only once. Computational results show that the resulting implementation of fast DWT is more than two times faster than the existing DWT library, and fast DWT significantly accelerates SPIRiT based MRI reconstruction.

## Introduction



Magnetic Resonance Imaging (MRI) is a powerful diagnostic modality for cardiovascular disease. State-of-the-art MRI reconstruction techniques such as  $l_1$ -regularized Self-consistent Parallel Imaging Reconstruction (SPIRiT) utilize compressed sensing to reduce data acquisition requirements.

The SPIRiT method, first proposed in 2007 [1], enforces calibration consistency between every point on the  $k$ -space grid, and its entire neighborhood across all coils [2].

The cost function of  $l_1$ -regularized SPIRiT is:

$$f(x) = g(x) + \mu \|\psi x\|_1$$

● Sampled Point  
● Evaluated Point

where  $g(x) = \|Fx - y\|_2^2 + \lambda \|Gx - x\|_2^2$ ,  $f$  is the  $k$ -space sampling operator,  $g$  is the SPIRiT self-consistency operator (see the diagram above), and  $\psi$  is a sparse transform operator (DWT in this case) plus 2D inverse FFT.  $x$  is the reconstructed  $k$ -space data and  $y$  is the acquired  $k$ -space data.  $\lambda$  and  $\mu$  are the tuning parameters.

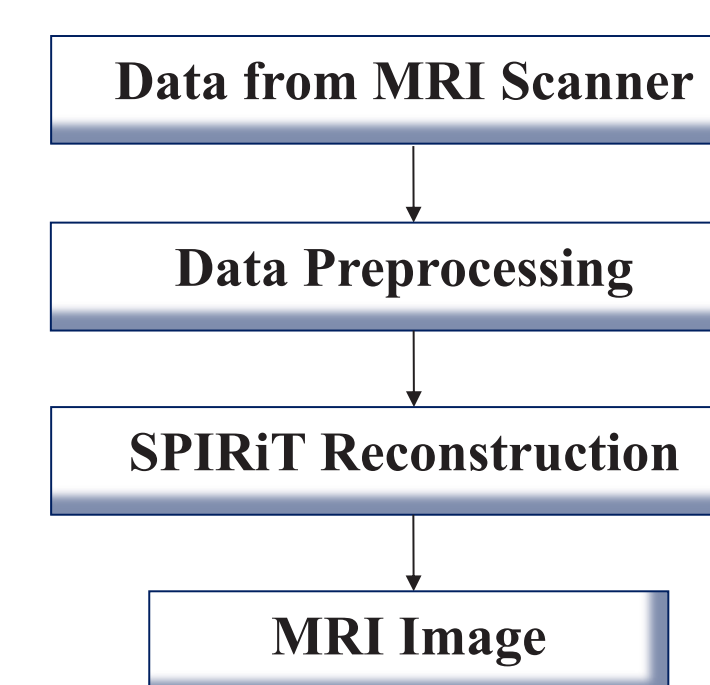
The cost function  $f(x)$  is minimized using the nonlinear conjugate gradient method with the following algorithm:

- Perform a line search:  
while  $f(m_k + t\Delta m_k) > f(m_k) + at \cdot \text{Real}(g^* k \Delta m_k)$   $\{t = \beta t\}$
- Calculate the steepest direction:  $g_{k+1} = \nabla f(m_{k+1})$ ;
- Upgrade the conjugate direction:  $\Delta m_{k+1} = -g_{k+1} + \gamma \Delta m_k$ ;

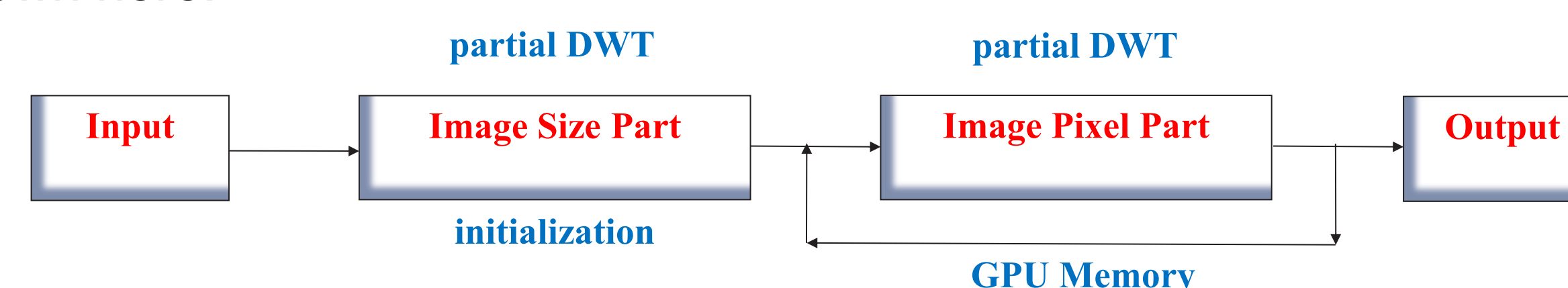
The nonlinear Conjugate Gradient algorithm involves a line search. Therefore, the cost function  $f(x)$  must be evaluated multiple times in every iteration. As a result, the DWT must be calculated for multiple times in every iteration of SPIRiT.

## GPU Implementation

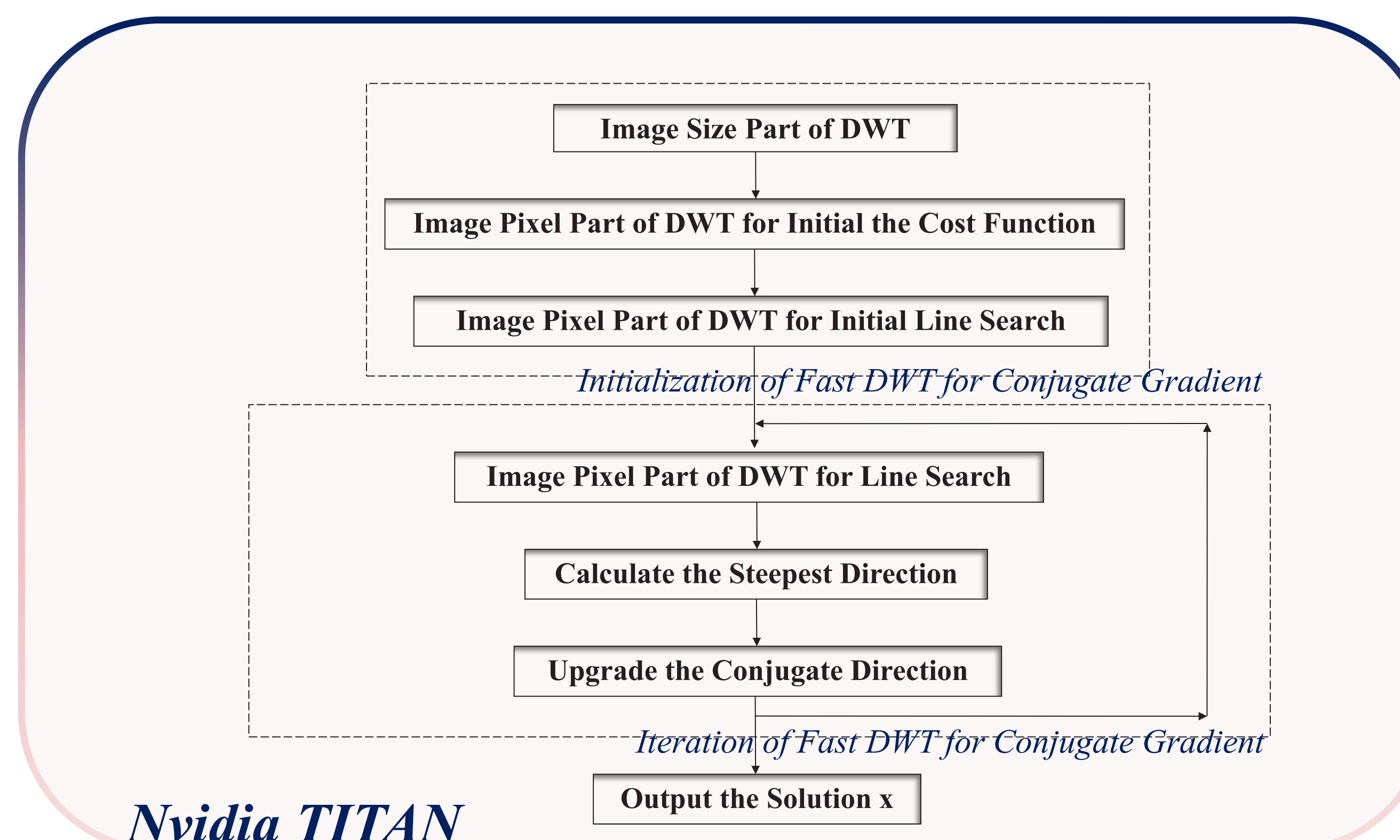
A GPU implementation of SPIRiT MRI reconstruction usually consists of four parts: receiving data from the MRI scanner, data preprocessing, SPIRiT reconstruction and MRI image output/storage. In SPIRiT, there is a constrained convex optimization problem to be solved, which requires solution of a symmetric and positive-definite linear system.



In routine implementations of DWT, the computation can be divided into two parts: one part depends on the pixel values, while the other depends only on the size of the image. In general DWT libraries, the input image is unknown and both parts are calculated in each execution of the DWT. However, in a typical dynamic series of MR images consisting of multiple temporal frames, the size of each image is identical; this property is common to most series of MR images. Taking advantage of this property, we propose to a GPU fast DWT algorithm (fDWT) to reconstruct a dynamic series of MRI images using a GPU as shown here:



From the figure above we can see that computations dependent on image size alone can be performed only in initialization, and the image pixel part must be calculated after initialization. Meanwhile, there is accumulation by for-loop in calculating convolution; generally we take advantage of GPU memory to speed up this process.



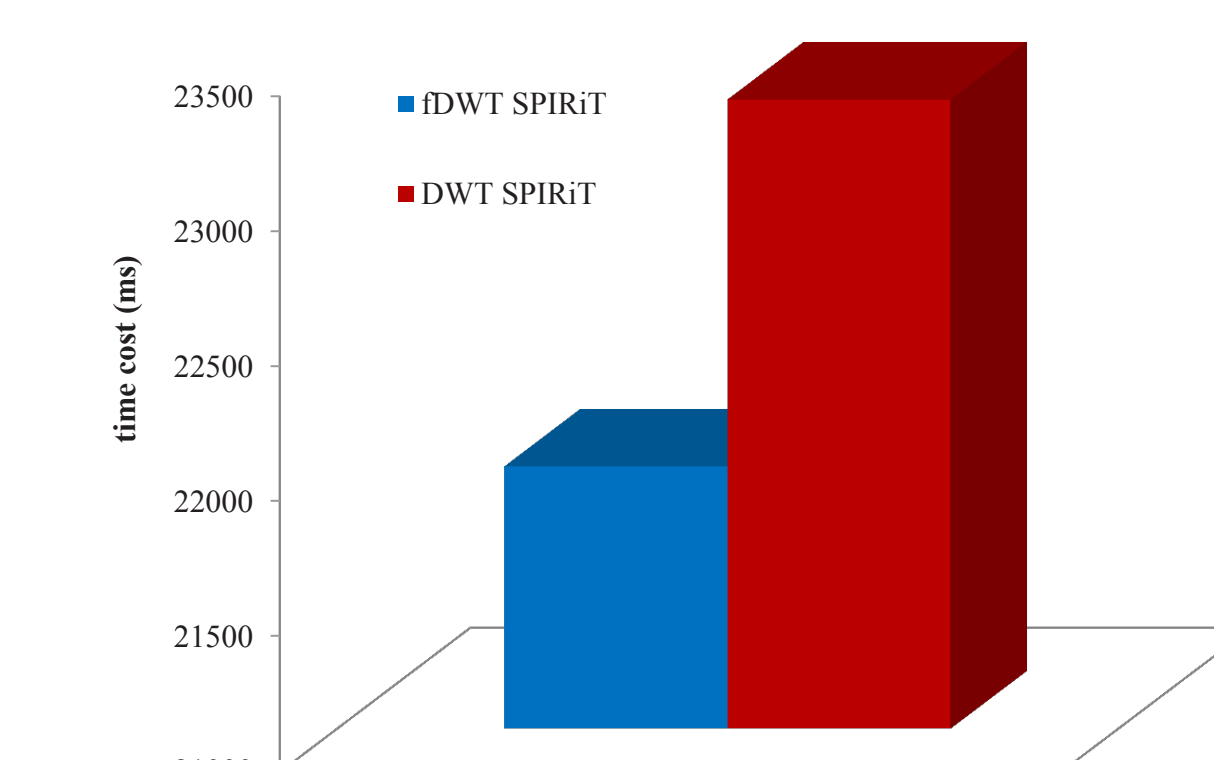
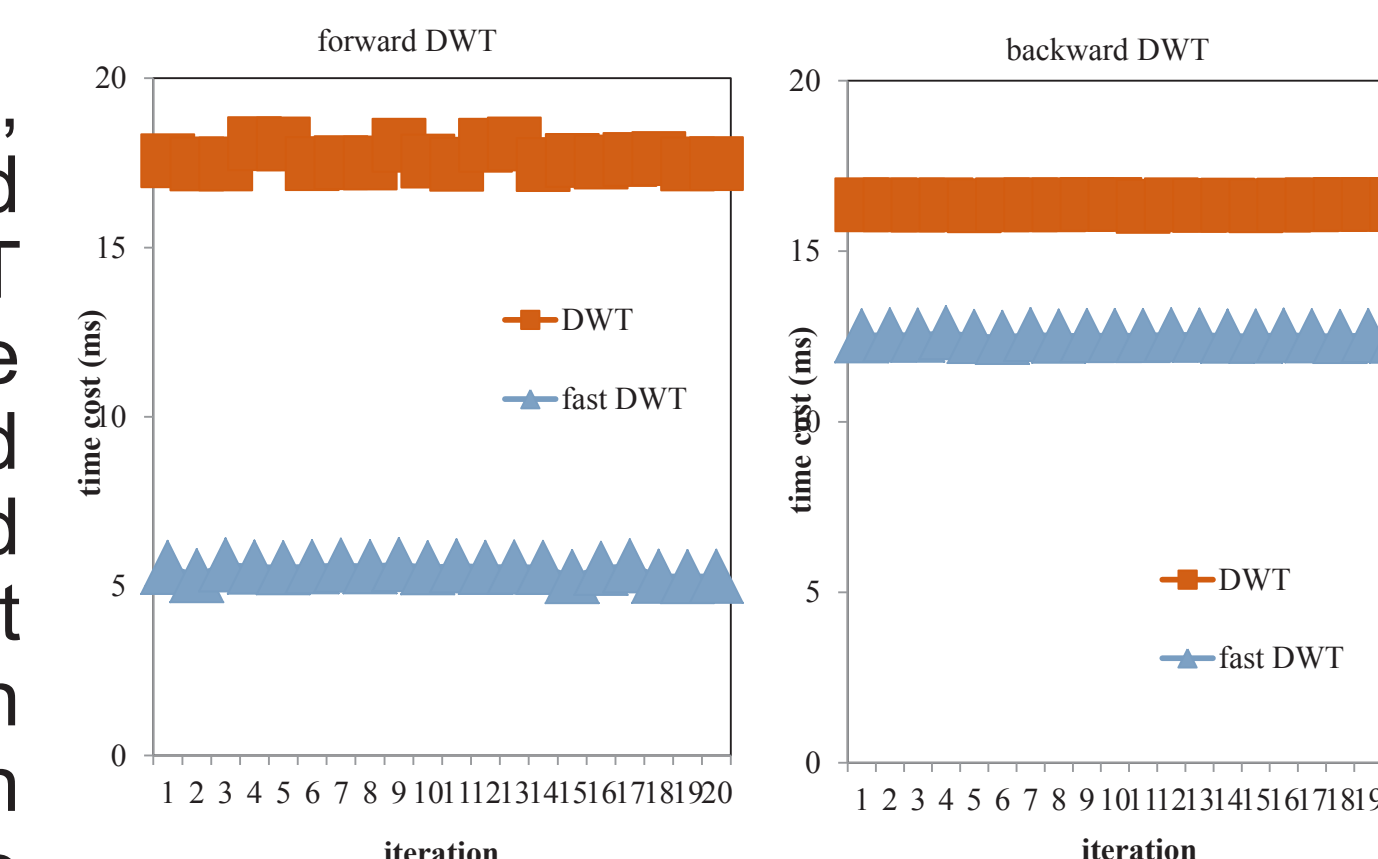
Nvidia TITAN

We implemented the GPU fDWT within the Conjugate Gradient algorithm for MRI reconstruction on an Nvidia TITAN. The image size part is calculated only during Conjugate Gradient initialization, and the image pixel part is calculated in every Conjugate Gradient iteration.

## Computational Results

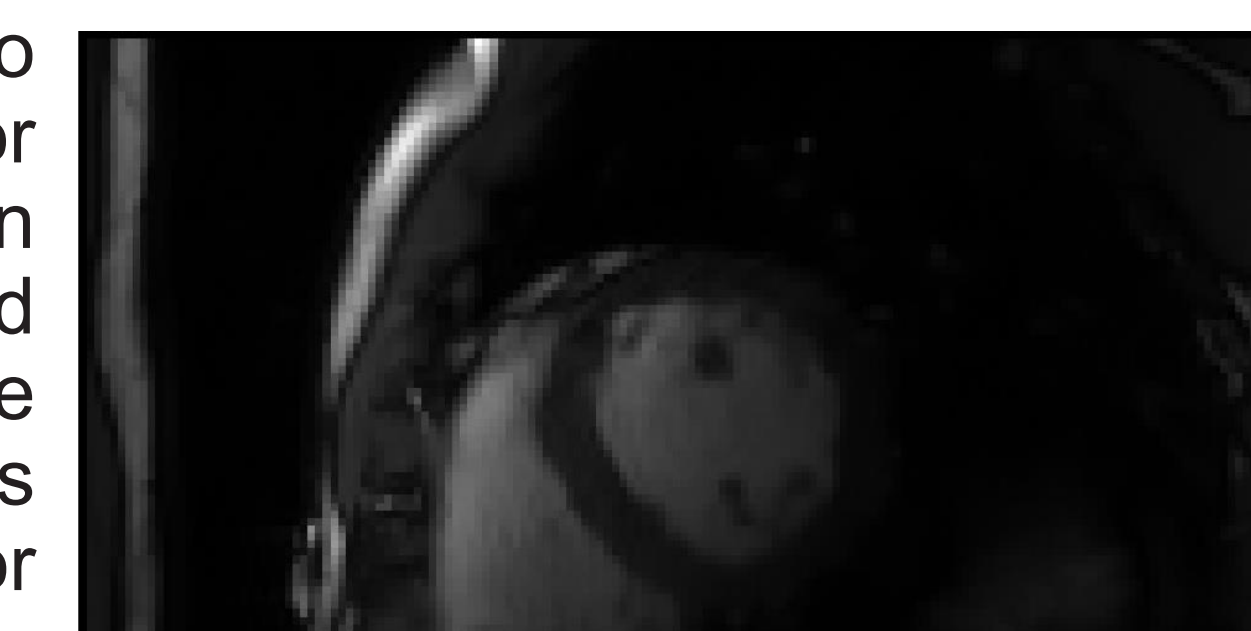
We implemented fast DWT for FISTA SPIRiT reconstruction on a GPU server with an Intel i7-3930K, 64 GB memory, and Nvidia TITAN with 6 GB memory. The performance of fast DWT was tested against the conventional DWT, and incorporated into the FISTA SPIRiT reconstruction for a  $96 \times 150 \times 16 \times 44$  Cardiac dataset.

In FISTA SPIRiT reconstruction, GPU fDWT was compared against the conventional DWT along with iterations: the time cost of forward DWT is plotted in the left figure, and backward DWT is plotted in the right figure. From the figures we can see GPU fDWT is more than two times faster than the conventional DWT at any number of iteration.



During FISTA SPIRiT reconstruction, the DWT must be calculated hundreds of times. The time cost of GPU fDWT based SPIRiT is compared against the conventional DWT based SPIRiT, and the time costs are plotted in the left figure. From the figure we can again see, GPU fDWT based FISTA SPIRiT is more than one second faster than that of the conventional DWT.

GPU fDWT was incorporated into FISTA SPIRiT reconstruction for Cardiac MRI on Gadgetron [3]. An example of the reconstructed images is plotted in the right figure with  $Fe=160$  and  $PE=80$ . It shows that GPU fDWT works well for FISTA SPIRiT reconstruction.



## Conclusion

GPU fDWT is more than two times faster than the conventional DWT for MRI reconstruction.

GPU fDWT significantly accelerates SPIRiT with compressed sensing.

GPU computing is the ideal platform for MRI reconstruction.

## References

- [1] Lustig, M., Donoho, D. and Pauly, J. M. Sparse MRI: The application of compressed sensing for rapid MR imaging. *Magnetic Resonance in Medicine*, 2007,58(6):1182-1195.
- [2] Lustig, M. and Pauly, J. M. SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary  $k$ -space. *Magnetic Resonance in Medicine*, 2010,64(2):457-471.
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This project is sponsored by NIH R01HL102450.