Accelerating Shallow Water Flow and Mass Transport Using Lattice Boltzmann Methods on GPUs

Kevin Tubbs

Dell | Global HPC Solutions Engineering

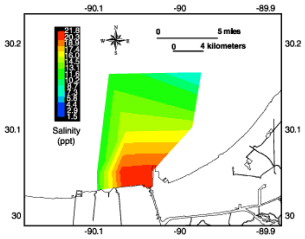
Introduction

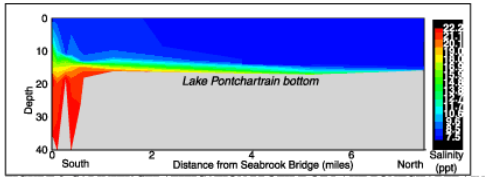
Background and Overview

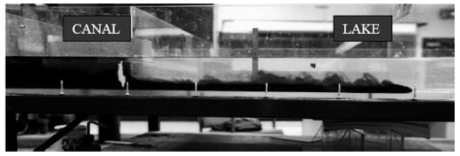
Governing Equations (PDE's) + Numerical Algorithm + HPC = Modeling Shallow Water flow and Mass Transport

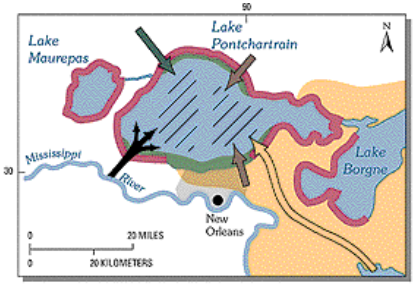
Role of CFD

- Improve Environmental Predictions
- Used to Improve Decision and Policy Making
- Test and Evaluate Preventative Methods









Dell Global HPC Solutions Engineering

Performance Analysis & Characterization

- Node level
- Cluster level

Power Consumption and Energy Analysis

Benchmark and Application Analysis

- Industry Standard Benchmarks & Apps: HPL, NPB, NAMD
- Focus Area Specific: CFD, Climate & Weather, Molecular Dynamics, etc.

Why Accelereyes ? → Programmability

Faster	Writing Kernels	**Using Libraries**
Slower	Instruction Set Optimization	Compiler Directives
	Time - Consuming	Easy-to-use

ACCELEREYES

Jacket: MATLAB® on the GPU

MATLAB Enabled — Great Science

NVIDIA TESLA CUDA — Great Power

ArrayFire

C, C++, Fortran for CUDA & OpenCL

Lattice Boltzmann Methods

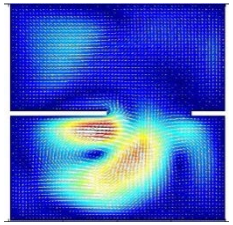
Lattice Boltzmann Method Overview:

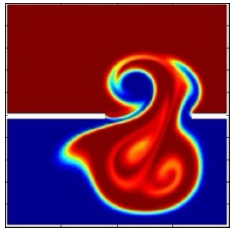
- Modeling Shallow Water and Mass Transport

Shallow Water Equation

$$\frac{\partial h}{\partial t} + \frac{\partial (hu_i)}{\partial x_i} = 0$$

$$\frac{\partial (hu_i)}{\partial t} + \frac{\partial (hu_i u_j)}{\partial x_i} = -g \frac{\partial}{\partial x_i}\left(\frac{h^2}{2}\right) + \nu \frac{\partial^2 (hu_i)}{\partial x_j \partial x_j} + F_i$$



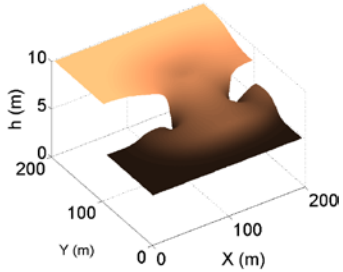


$$\frac{\partial f}{\partial t} + \mathbf{c} \cdot \nabla f = -\frac{1}{\tau}(f - f^{(eq)})$$

LB Numerical Formulation

Advection Dispersion Equation for Solute Concentration

$$\frac{\partial (hC)}{\partial t} + \frac{\partial (hu_j C)}{\partial x_j} = \frac{\partial}{\partial x}\left(D_{ij} h \frac{\partial C}{\partial x_j}\right) + F_C$$



Background: LBM

- Boltzmann equation $$\frac{\partial f}{\partial t} + \mathbf{e} \cdot \nabla f = -\frac{1}{\tau}(f - f^{(eq)})$$
- Particle distribution function $f(\vec{\mathbf{x}}, \vec{\mathbf{e}}, t) \quad g(\vec{\mathbf{x}}, \vec{\mathbf{e}}, t)$
- EDF

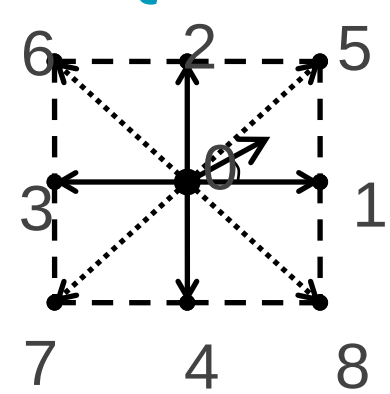
$$f_\alpha^{eq} = h\omega_\alpha \left(\frac{c_s^2}{e^2} + \frac{\mathbf{u} \cdot \mathbf{e}_\alpha}{e^2} + \frac{3}{2}\frac{(\mathbf{u} \cdot \mathbf{e}_\alpha)^2}{e^4} - \frac{1}{2}\frac{\mathbf{u} \cdot \mathbf{u}}{e^2} \right) \quad ,\alpha > 0 \qquad f_0^{eq} = h - \sum_{\alpha > 0} f_\alpha^{eq}$$

$$g_\alpha^{eq} = C\omega_\alpha \left(\frac{c_{s\alpha}^2}{e^2} + \frac{h\mathbf{u} \cdot \mathbf{e}_\alpha}{e^2} + \frac{3}{2}\frac{(h\mathbf{u} \cdot \mathbf{e}_\alpha)^2}{he^4} - \frac{1}{2}\frac{h\mathbf{u} \cdot h\mathbf{u}}{he^2} \right) \quad ,\alpha > 0 \quad g_\alpha^{eq} = hC - \sum_{\alpha > 0} g_\alpha^{eq}$$

- LBM Grid

$$\mathbf{e}_\alpha = \begin{cases} (0,0) & \alpha = 0 \\ e\left[\cos\frac{(\alpha-1)\pi}{4}, \sin\frac{(\alpha-1)\pi}{4}\right], & \alpha = 1,2,3,4 \\ \sqrt{2}e\left[\cos\frac{(\alpha-1)\pi}{4}, \sin\frac{(\alpha-1)\pi}{4}\right], & \alpha = 5,6,7,8 \end{cases}$$

D2Q9 LBM



Background: LBM (cont.)

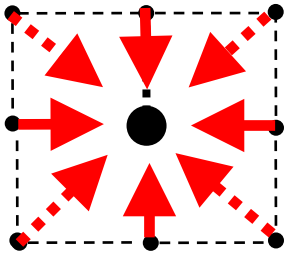
- Zeroth Moment
- First Moment

$$h = \sum_{\alpha} f_{\alpha}$$
$$hu_i = \sum_{\alpha} e_{\alpha i} f_{\alpha}$$

$$hC = \sum_{\alpha} g_{\alpha}$$
$$Chu_i = \sum_{\alpha} e_{\alpha i} g_{\alpha}$$

- LBM evolves in a two-step process
  - Step 1. Collision

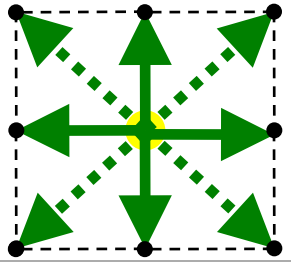
$$f_{\alpha}\left(\mathbf{x},\hat{t}\right) = f_{\alpha}\left(\mathbf{x},t\right) - \frac{1}{\tau}\left(f_{\alpha}\left(\mathbf{x},t\right) - f_{\alpha}^{eq}\left(\mathbf{x},t\right)\right) + \frac{\Delta t}{6e^2} e_{\alpha i} F_i\left(\mathbf{x},t\right)$$



Collide

  - Step 2. Streaming

$$f_{\alpha}\left(\mathbf{x}+\mathbf{e}_{\alpha}\Delta t, t+\Delta t\right) = f_{\alpha}\left(\mathbf{x},\hat{t}\right)$$

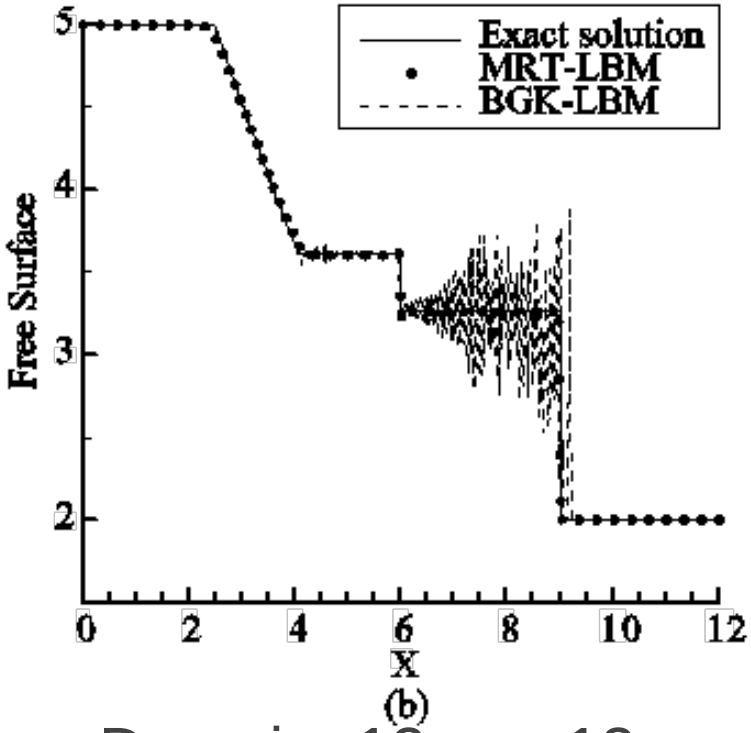


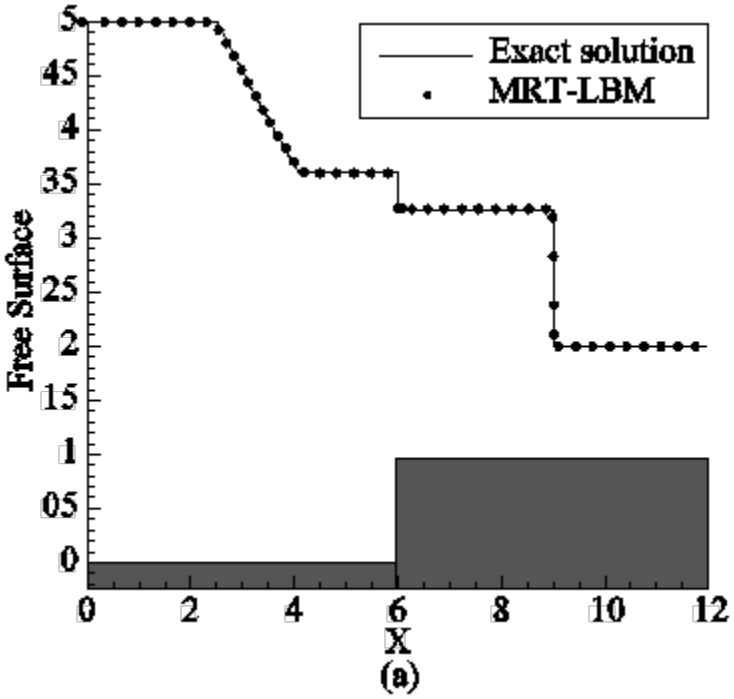
Stream

Results:

1) Model Validation
2. Simulation Results

2D – Dam Break: Backward Facing Step

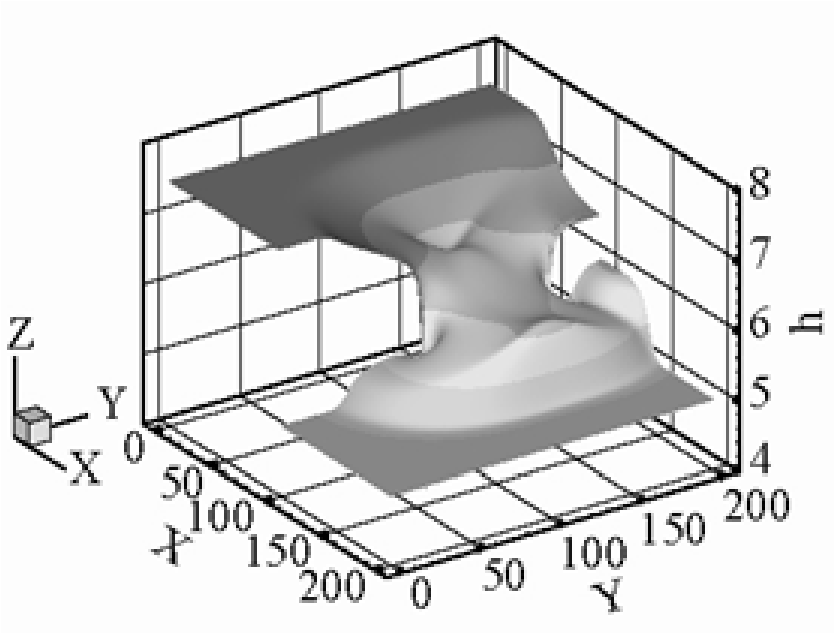




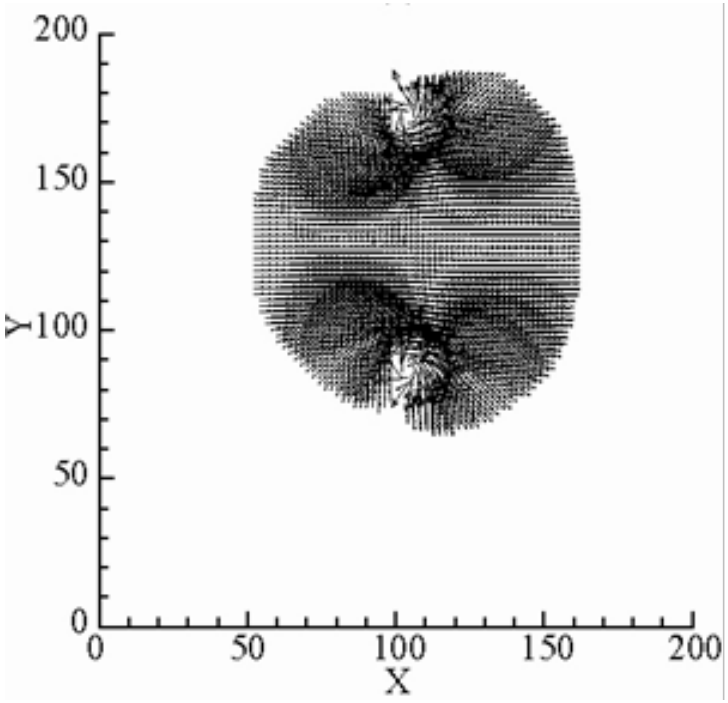
- Domain: 12 m x 12 m
- Dam Break Initial Conditions
  - Upstream: water depth 5 m
  - Downstream: water depth 1m
- BC: Free – Slip at walls for flow

t = 0.5 s

2D Partial Dam Break



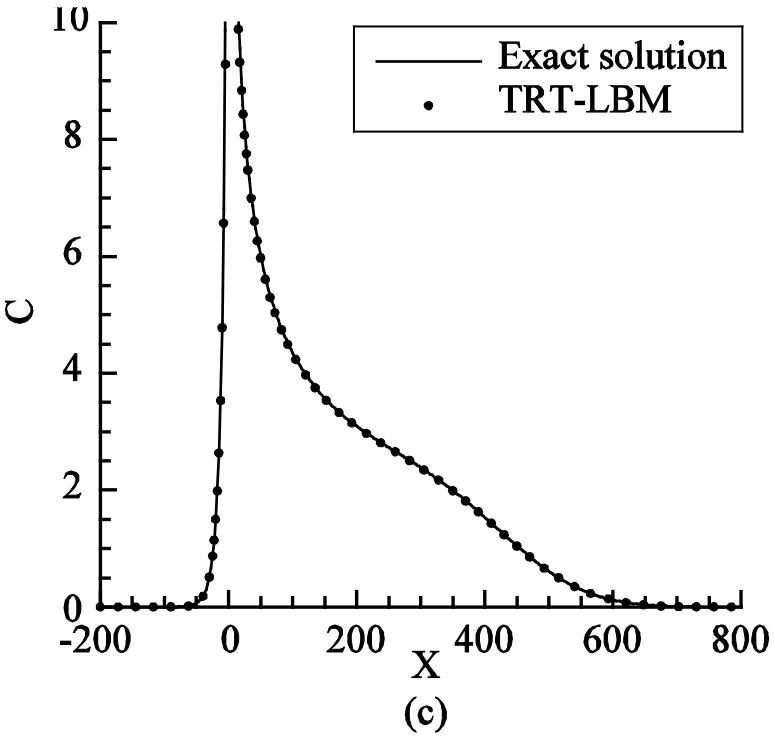
Water Depth Surface t = 7.2 s

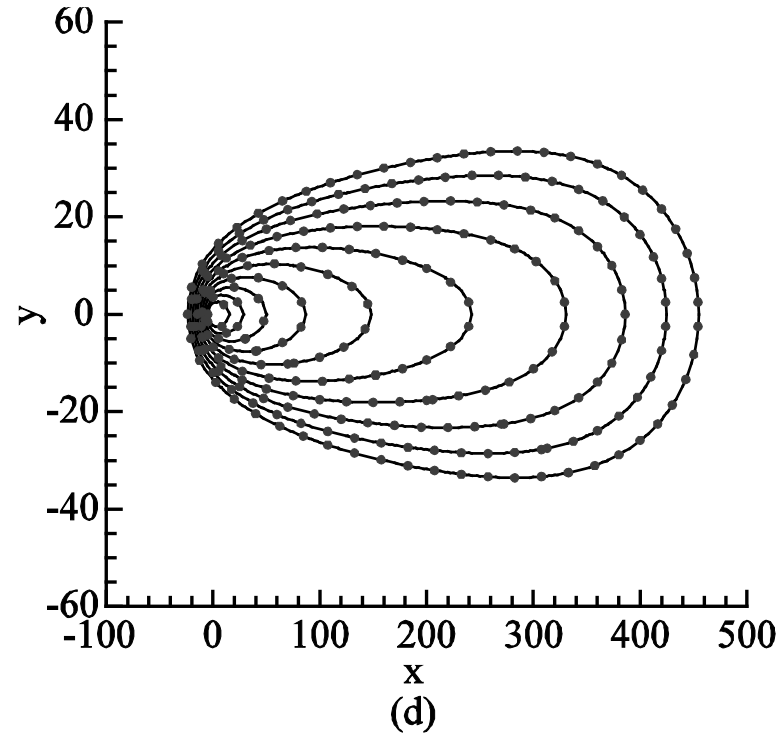


Velocity Vector Field t = 7.2 s

- Domain: 200 m x 200 m
- Dam Break Initial Conditions
  - Upstream: water depth 10m
  - Downstream: water depth 5m
- BC: Free – Slip at walls for flow, Impermeable for concentration

2D – Continuous Release Mass Transport





- Problem Setup
  - Domain: Infinite Shallow Water Pool
  - Constant Water Depth 1 m

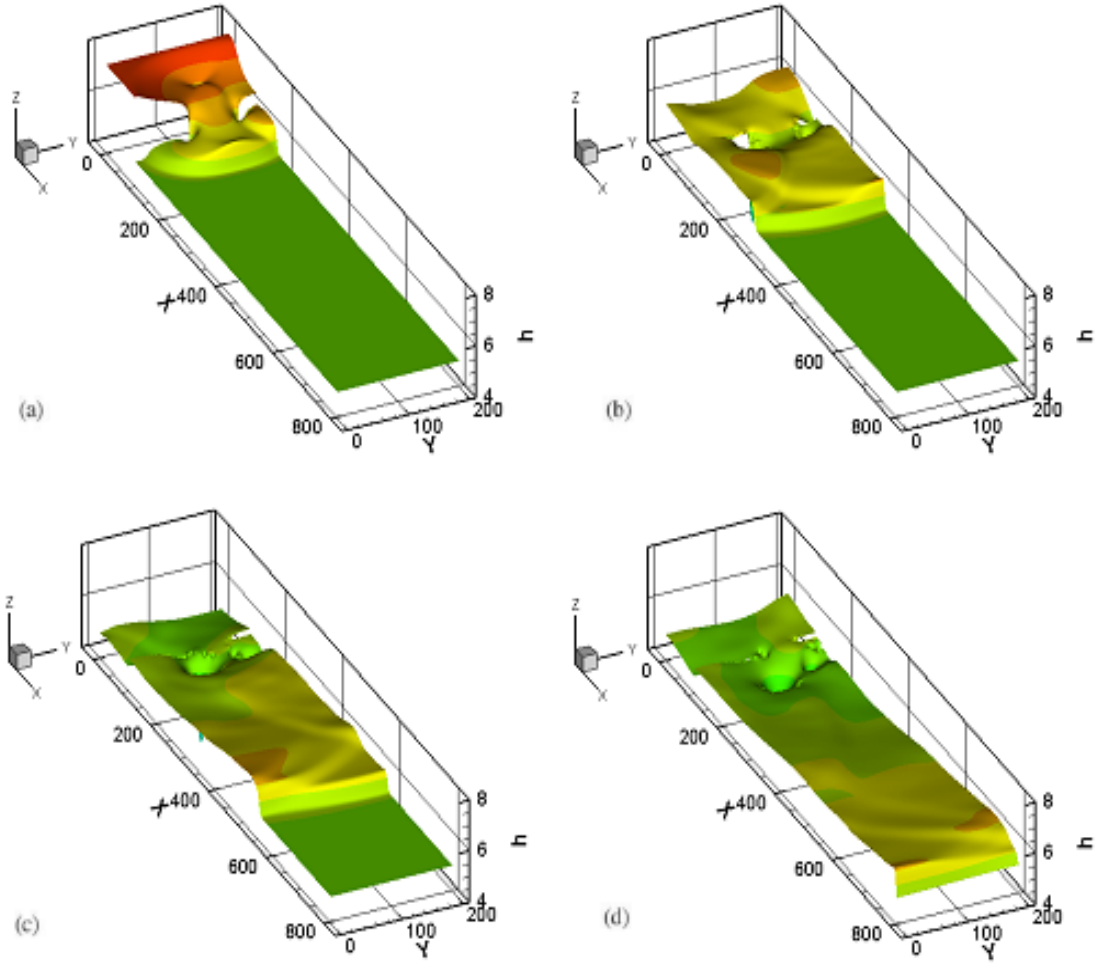
$$\tau_s = 1/2 + \left(12\left(\tau_A - 1/2\right)\right)^{-1} \qquad c_{sij}^2 = \frac{hD_{ij}}{\Delta t\left(\tau_A - 1/2\right)}$$

Velocity Anisotropic Dispersion

$$D_{xx} = h|\mathbf{u}|\kappa_L \qquad D_{yy} = h|\mathbf{u}|\kappa_T \qquad \kappa_L / \kappa_T = 10$$

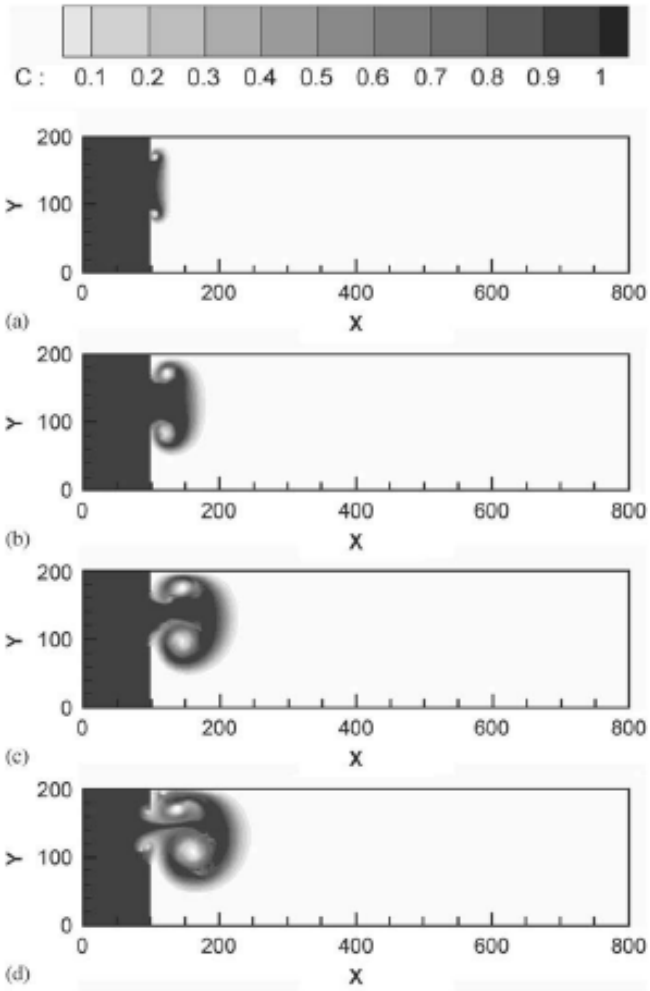
2D – Dam Break with Mass Transport

- Domain: 800m x 200m
- Dam Break with Transport Initial Conditions
  - Upstream: water depth 10m
  - Downstream: water depth 5m
  - BC: Free – Slip at walls for flow
- Time Snap shots
  - t = 10 s
  - t = 30 s
  - t = 60 s
  - t = 90 s



2D – Dam Break with Mass Transport

- Domain: 800 m x 200 m
- Dam Break with Transport Initial Conditions
  - Upstream: water depth 10m , Concentration 0.7
  - Downstream: water depth 5m, Concentration 0.2
  - BC: Free – Slip at walls for flow, Impermeable for concentration
- Time Snap shots
  - t = 10 s
  - t = 30 s
  - t = 60 s
  - t = 90 s



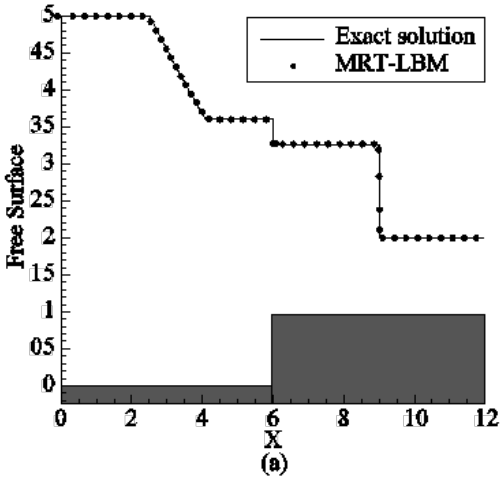
$$D_{ij} = \left(k_L |\mathbf{u}| \delta_{ij} + (k_L - k_T) \frac{u_i u_j}{|\mathbf{u}|} \right) \frac{h\sqrt{g}}{C_z}$$

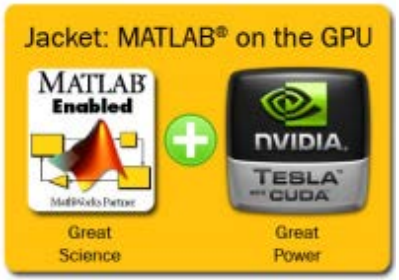
$$k_L = 5.93$$

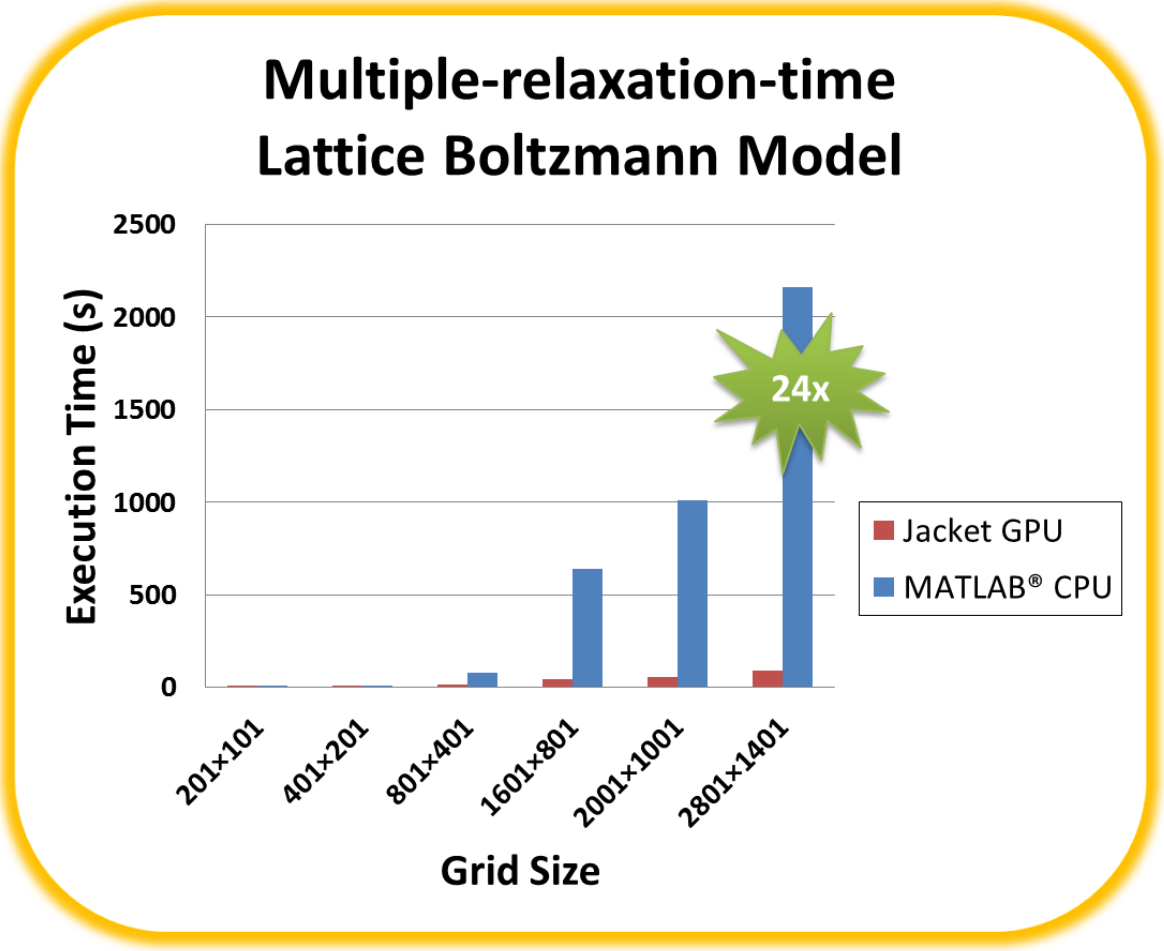
$$k_T = 0.23$$

Performance

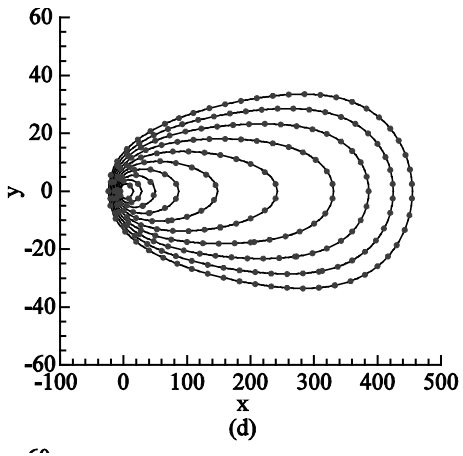
2D – Dam Break: Backward Facing Step

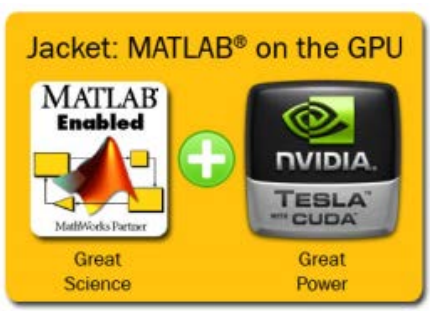


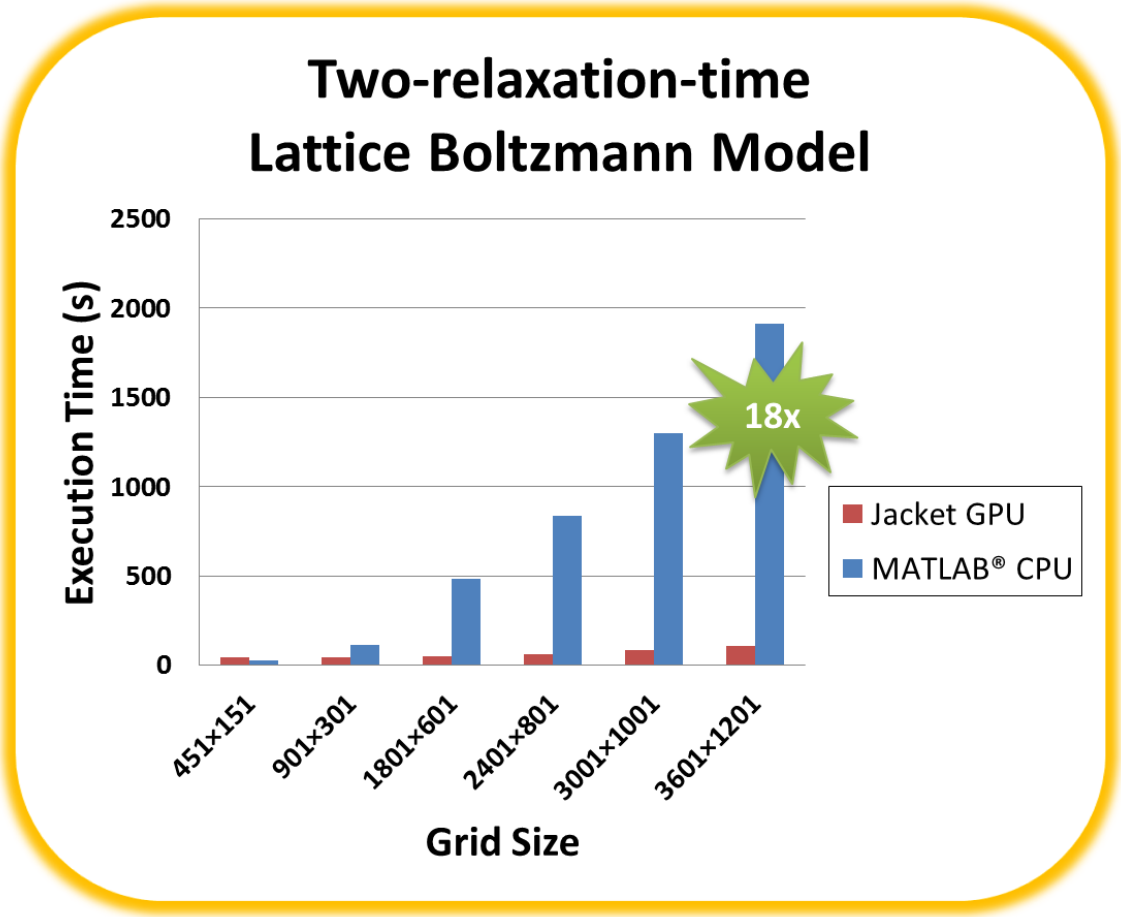




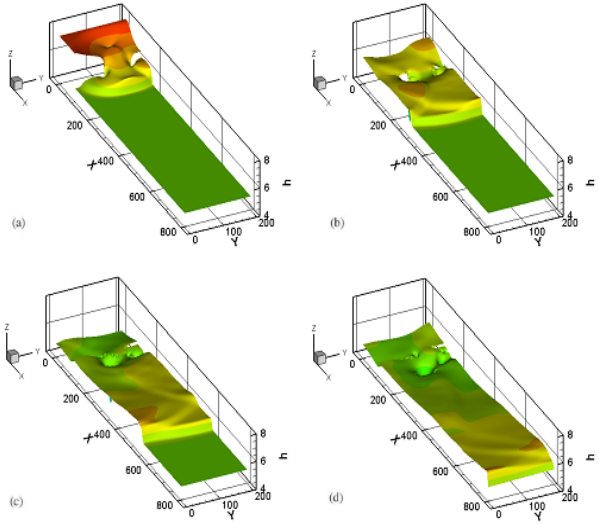
2D – Continuous Release Mass Transport

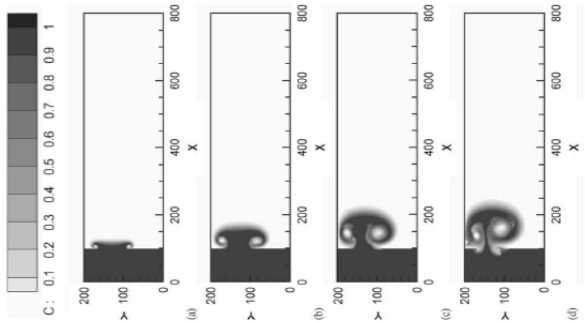


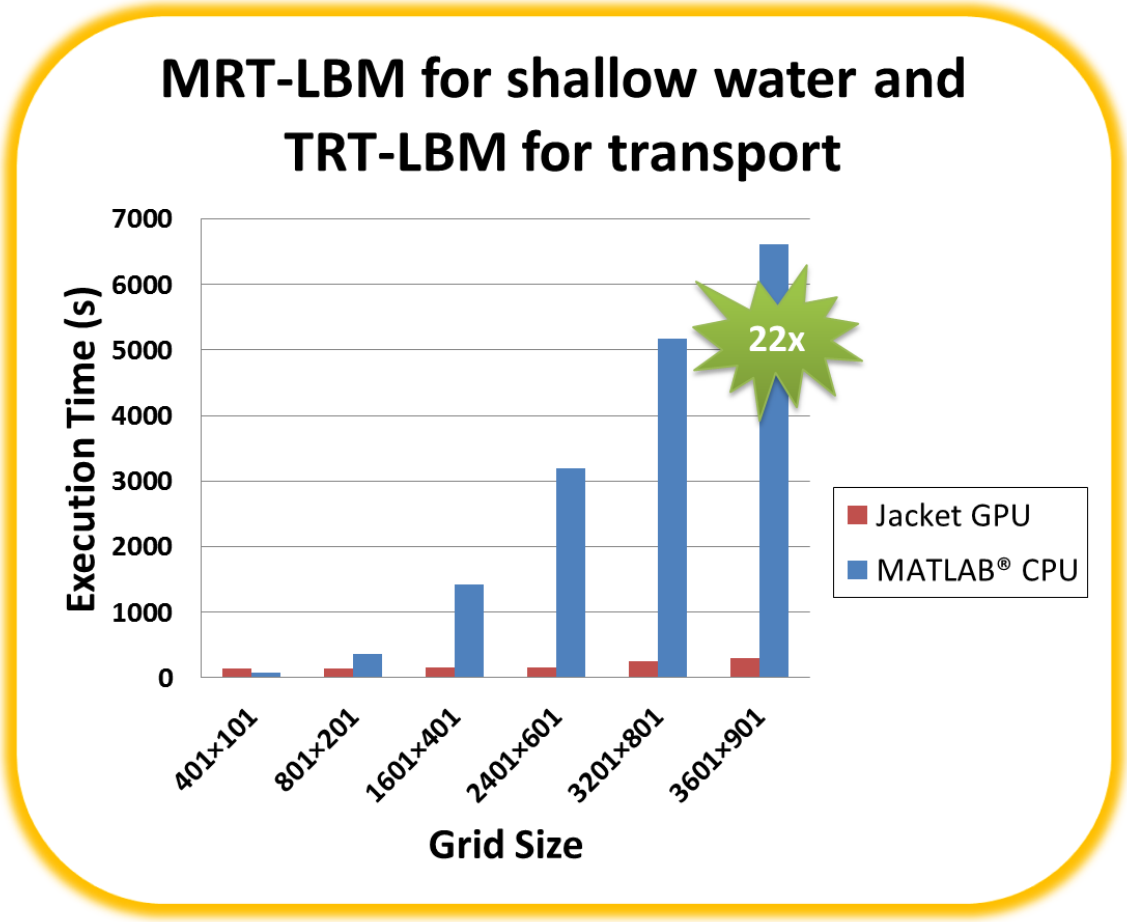




2D – Dam Break with Mass Transport





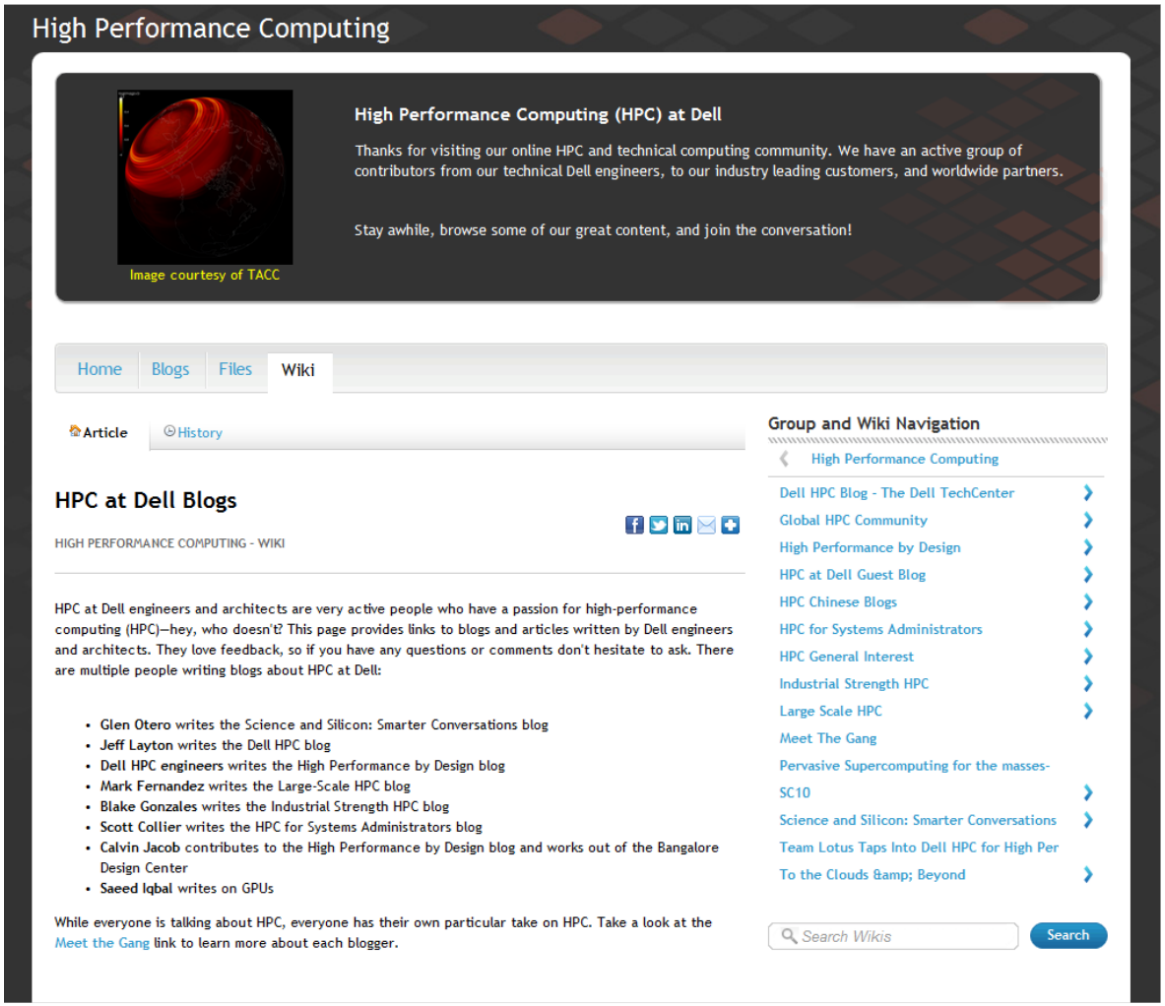


Summary

- LBM is a flexible algorithm for various CFD simulations
- Tools from Accelereyes allow rapid development and validation of LBM based CFD algorithms
- The GPU accelerated LBM algorithm achieved maximum of 24X speedup on GPU based architectures.
- The GPU parallel performance depends on size of simulation.

Resources

- Blogs
- Whitepapers
- HPC Advisor Online



- www.dell.com/gpu
- www.dell.com/hpc
- www.hpcatdell.com
- www.DellHPCSolutions.com
- http://www.hpcadvisorycouncil.com/best_practices.php

Related Work

Publications, Proceedings & Presentations:

Publications

Tubbs, K. R., and F. T.-C. Tsai, GPU Accelerated Lattice Boltzmann Model for Shallow Water Flow and Mass Transport. Int. J. Numer. Meth. Eng (2010) DOI: 10.1002/nme.3066

http://blog.accelereyes.com/blog/2010/12/01/lattice_boltzmann_model/

Tubbs, K. R., and F. T.-C. Tsai, Multilayer Shallow Water Flow using Lattice Boltzmann Model with High Performance Computing. Advances in Water Resources, 32(12), 1767-1776, 2009.

Presentations

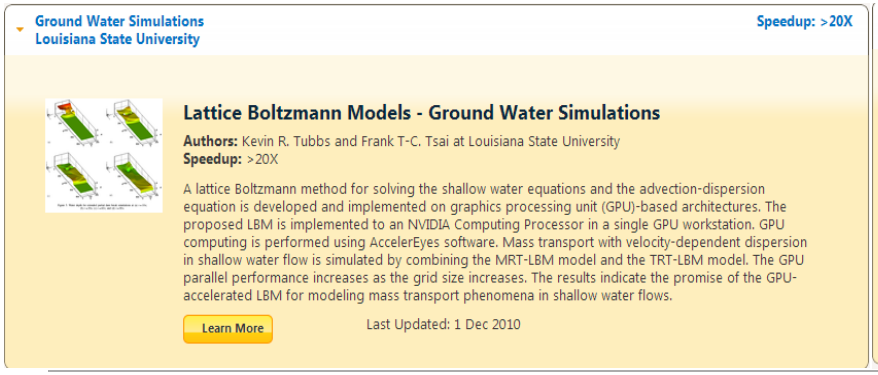
Tubbs, K. R., and F. T.-C. Tsai, A GPU Accelerated Lattice Boltzmann Model for Shallow Water Flow and Mass Transport, Sixth International Conference for Mesoscopic Methods in Engineering and Science (ICMMES), Guangzhou City, Guangdong Province, China, July 13-17, 2009.

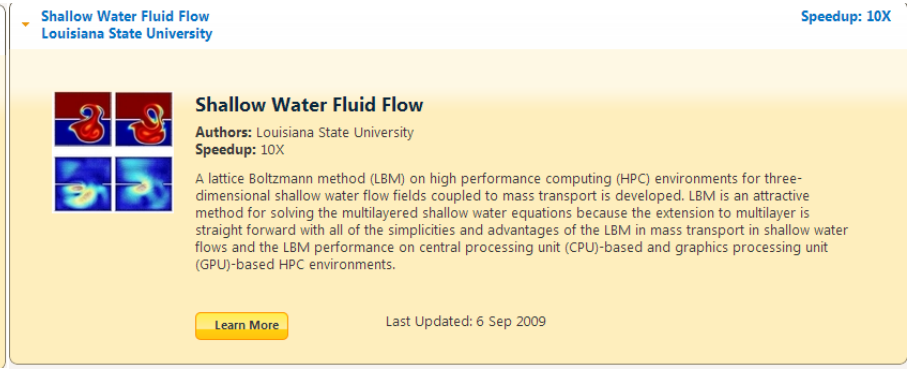
Proceedings

Tubbs, K. R., and F. T.-C.. Tsai. Simulation of Multilayer Shallow Water Fluid Flow Using Lattice Boltzmann Modeling and High Performance Computing. *ASCE/EWRI World Water & Environmental Resources Conference, Kansas City, Missouri, May 17-21, 2009.*

Tubbs, K. R., and F. T.-C.. Tsai, C. White, G. Allen. Lattice Boltzmann Modeling Using High Performance Computing for Shallow Water Fluid Flow and Mass Transport, American Geophysical Union 2008 Fall Meeting, San Francisco, CA, Dec 15-19, 2008.

http://www.accelereyes.com/success_story/shallow_water_fluid_flow.php





Questions