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Solving interval linear system of equations using GPU computing

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Abstract

Considered is linear equation set **A**x = **b** with interval matrixes **A, b**. Solutions are items of Θ_{tol} (**A, b**). Let Θ_{tol}(**A**, **b**(z)) = {x : **A**x =(1 + z)b)}, z* = inf {z : Θ_{tol} (A, b(z)) $\neq$ Ø} be. Items of set Θ_{tol} (A, b(z*)) is termed as pseudosolutions. Existence of pseudo-solution for all interval algebraic linear equation set is proved in the paper, the way of pseudo-solution retrieving by solving the corre-sponding linear programming problem is suggested. It is necessary com-putation guaranteeing sufficient accuracy well above standard data types of programming languages because of result of obtained problem degen-eracy. Simplex method coupled with errorless rational-fractional compu-tation gives effective solution of the problem. Coarse-grained parallelism for distributed computer systems with MPI is instrument of realization. CUDA C software engineering is suggested for errorless rational-fractional calculations and small-grained parallelism.

Pseudo-solution of interval linear equation set

Let **A**x = **b** is given linear system of equation, where elements of the matrices **A** and **b** are intervals $\mathbf{a}_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}]$, $\mathbf{b}_j = [\underline{b}_j, \overline{b}_j]$, i,j = 1, 2,...,n.
For given system of equations we construct parameterized family of system of equations Ax = b(z) with modified right part **b**(z) = [b — z |b| , b + z |b|] , z > 0.
Let z* = inf {z : Θ_{tol} (A, b(z)) $\neq$ Ø}. "Pseudo-solution" of basic system Ax = b we call inner points of tolerable set Θ_{tol} (A, b(z*)).
Correctness of introduced definition proves theorem.
Theorem 1. *For any interval system of equations* **A**x = **b** *for all* z > 1 *set* Θ_{tol}(A, b(z)) $\neq$ Ø.

Way to compute the pseudosolution

Way to find pseudo-solution of system of equation **A**x = **b** gives
Theorem 2. *Exist solution* x^+, x^- *belonging* $\mathbf{R}^n$, *and* z* *belonging* **R** *of linear programming task*

$$z \to \min_{x^+, x^-, z},$$

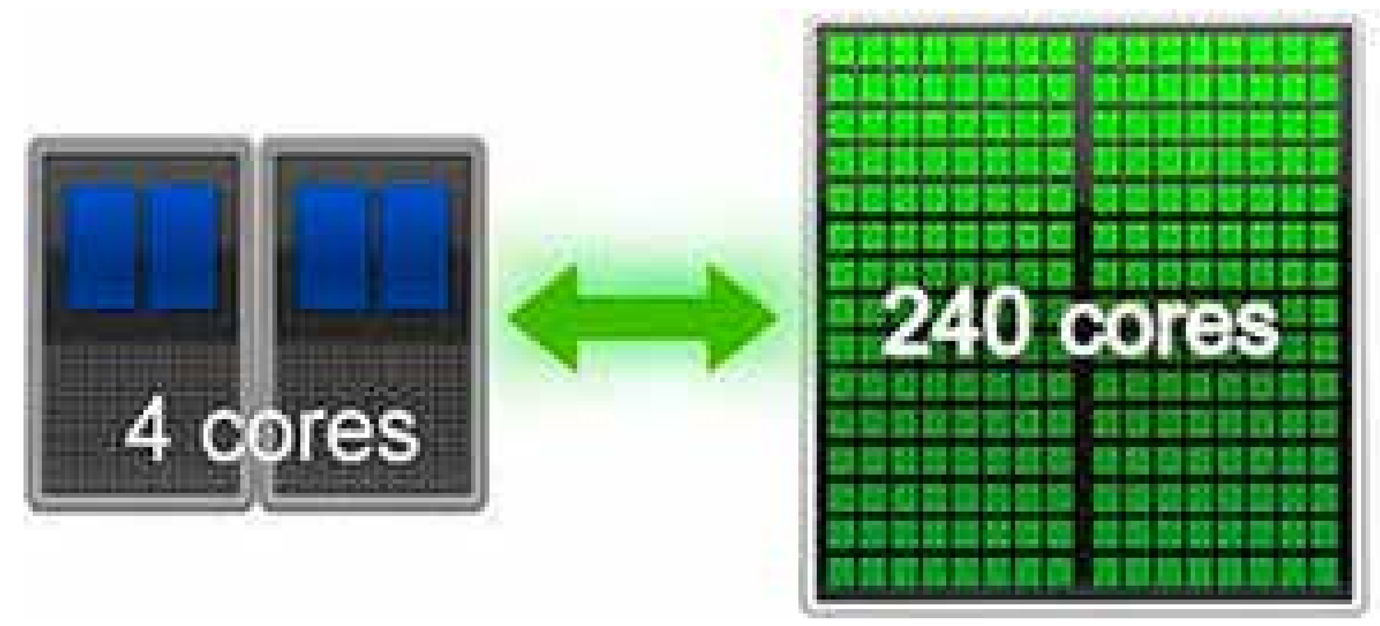
$$\sum_{j=1}^{n} (\underline{a}_{ij} x_j^+ - \overline{a}_{ij} x_j^-) \geq \underline{b}_i - z\,|\underline{b}_i|, \quad i = 1, 2, \ldots, n,$$

$$\sum_{j=1}^{n} (\overline{a}_{ij} x_j^+ - \underline{a}_{ij} x_j^-) \leq \overline{b}_i + z\,|\overline{b}_i|, \; i = 1, 2, \ldots, n,$$

$$x_j^+, x_j^-, z \geq 0, \quad j = 1, 2, \ldots, n,$$

besides $x^* = x^+ - x^-$ *is pseudosolution of system* **A**x = **b** .

It should be noted that problem is high degenerates.
There for quality implementation requires careful elaboration of some moments. First of all it is necessary to provide fine exactness of the computing to prevent simplex-method's cycling. Such anti-cycling techniques for simplex introduced in [21]. Exact computations should be extremely fast to solve tasks within comfortable time limits. Then linear task should be carefully decomposed to use modern processors multicore/multithread and cluster architectures possibilities.



Necessary precision of the calculations

Proposed exact computation tool was developed earlier in South Ural State University as C++ classes overlong and rational (almost so useful as standard C++ data types, but without their range and precision limitations) [16, 17].
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Brief review of the up to date version of the classes overlong and rational is given in [5]. Volatility and technical support for all memory operations are incapsulated in special memory handle class MemHandle in other hand all basic arithmetic operations are incapsulated in ArifRealization class (see listing 1).

```
class  overlongNM { private :
static   ArifRealization realization;
private :
MemHandle mhandle;
.....................
public: inline int32 size() const {return leng;}//length public:
inline   int32   sign()   const   {return  sgn ;} //sign
.....................
// addition
template<typename Type>friend   const  overlongNM operator+ (const
overlongNM &num,Type v) {overlongNM  rez(num);   return   (rez+=v);}
friend   const  overlongNM operator+
(const  overlongNM&, const  overlongNM&);
.....................
}
```

Listing 1: Fragment of `overlong class`

So overlong object contains MamHandle object for memory operations and static ArifRealization link to basic arithmetic operation realization. All basic arithmetic operation corresponding methods of ArifRealization class. Sample for the addition operation demonstrated in the listing 2.

```
void  overlongNM::add(const overlongNM &alpha, const overlongNM& beta)
{
d_t   carry ;
const  overlongNM& a=(alpha.size()> = beta.size())?   alpha: beta;
const  overlongNM& b=(alpha.size()> = beta.size())?   beta : alpha;
int32 LA=a. size(),LB=b.size(),sg=alpha.sgn,newleng;//LA>=LB
ArifRealization::add(a.mhandle.getptr(), LA, b.mhandle.getptr(),LB,
mhandle.providetmpptr(LA,1), new leng, carry);
mhandle.settm pasptr() ;
if (carry) mhandle.safesetvalue(LA,carry); leng=newleng; sgn=sg ;
}
```

Listing 2: Addition operation managment

Computing experiment

Computing experiment was performed on the Intel(R) Core(R) i7-950 processor 3.06GHz with 6 GB RAM and Nvidia(R) GTX-460 with 1Gb GDDR5. Code was compiled with Visual C++ 2011 compiler. Model task was solving interval linear system of equations with Hilbert matrix and point right part b = [1,1/2,..., 1/(n - 1), 1/n]^T. The minimal extension of the right part of the system dependence on parameter 5 (parameter z*) corresponding to pseudo-solution when fixed *n* = 20 given in the table

δ	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
z*	0.81	0.389	0.1	0.025	0.0062	0.0017

Table 2 contains the results for different model task size with fixed *n*

n	10	20	50	100
time	0.46 sec	7.73 sec	7.39 min	15.1 hours

Conclusion

Suggested CUDA C software engineering for errorless rational-fractional calcula-tions and small-grained parallelism enable efficiently perform of them. Simplex method coupled with errorless rational-fractional computation gives ef-fective solution of the problem. Coarse-grained parallelism for distributed computer systems with MPI is the suitable instrument for realization.

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