FUZZY LOGIX™

Intra-day Risk Management

Using

Parallelized Algorithms on Graphics Processing Units (GPUs)

Presented By

FUZZY LOGIX™

TANAY

Why Market Risk Management?

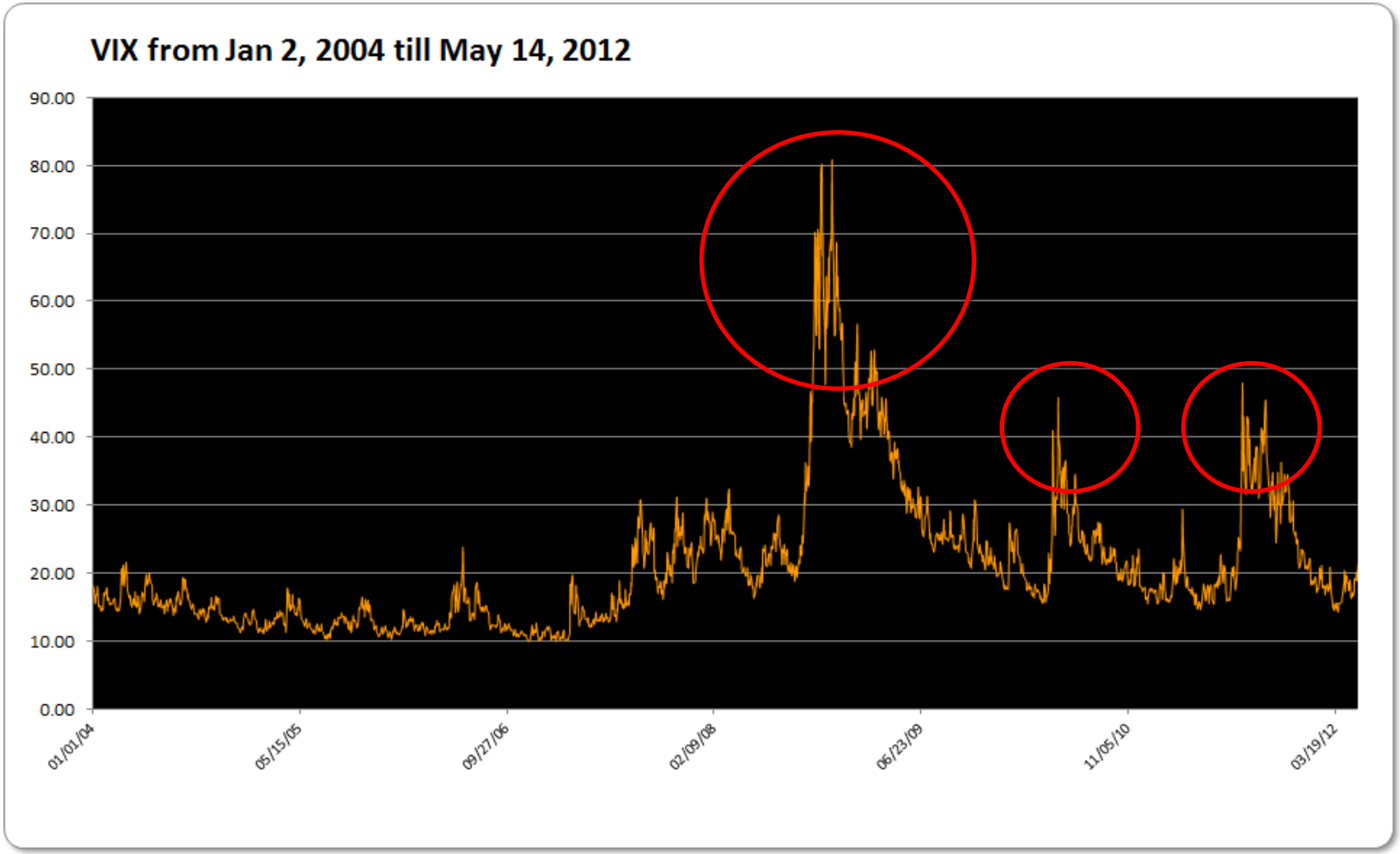
FUZZY LOGIX

- **Heard of these? MF Global, Bears Stearns, and most recently the English Whale and J P Morgan**
- **Financial Institutions need to understand the Mark-to-Market Value**
  - Market Values are not static
  - Is there an orderly market? Should we even try to establish orderly markets?
  - Attempts are being made by regulators and legislators to force markets to behave in an orderly manner (MiFID). Not sure whether these type of things ever work?
- **Implication of unpredictable market volatility in recent times**
  - Intra-day risk management has become even more important for proactive margin management and closeout risk assessment

TANAY

Volatilities have been high in recent times

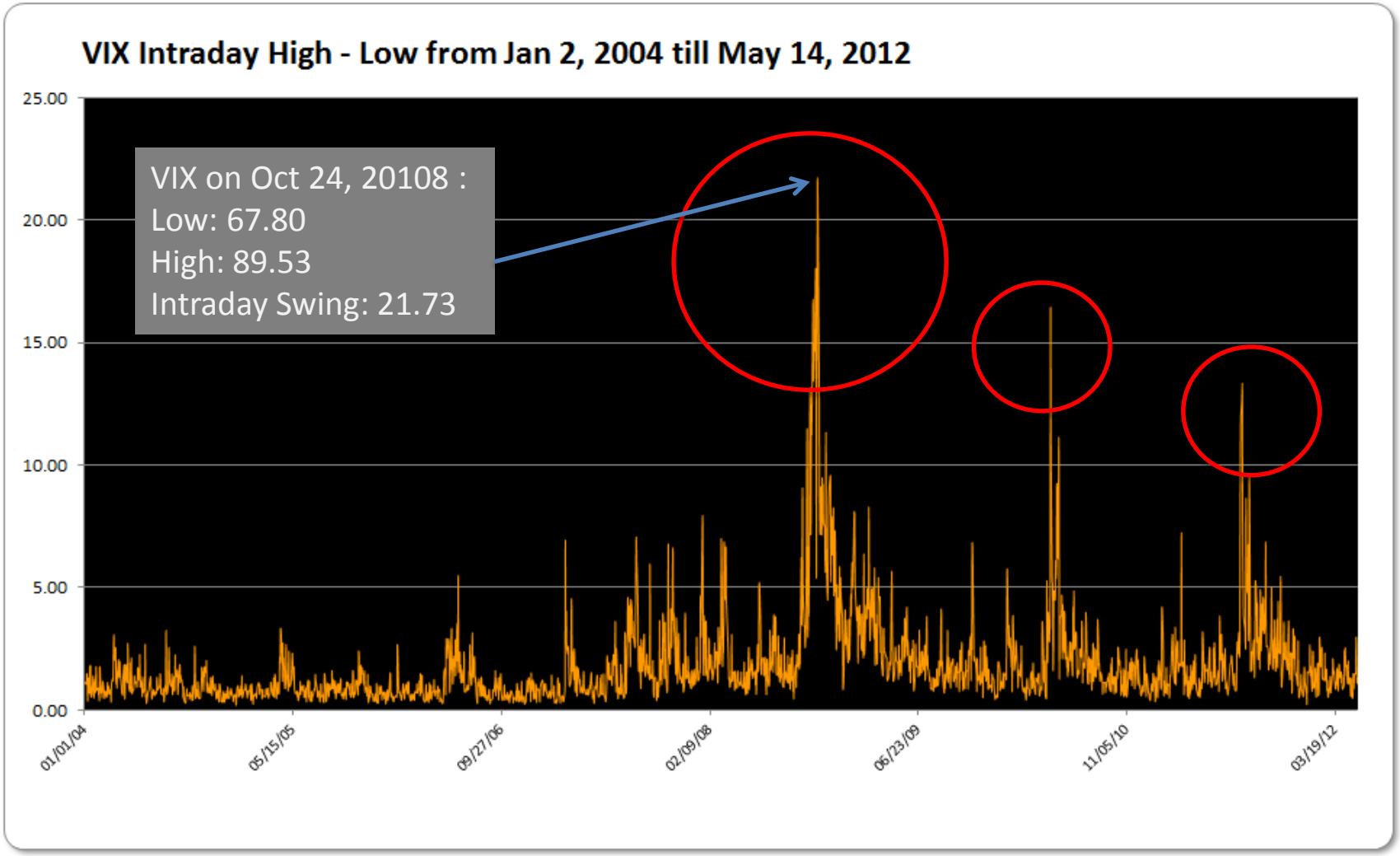
FUZZY LOGIX



TANAY

Intraday swings in VIX have also been quite high

FUZZY LOGIX



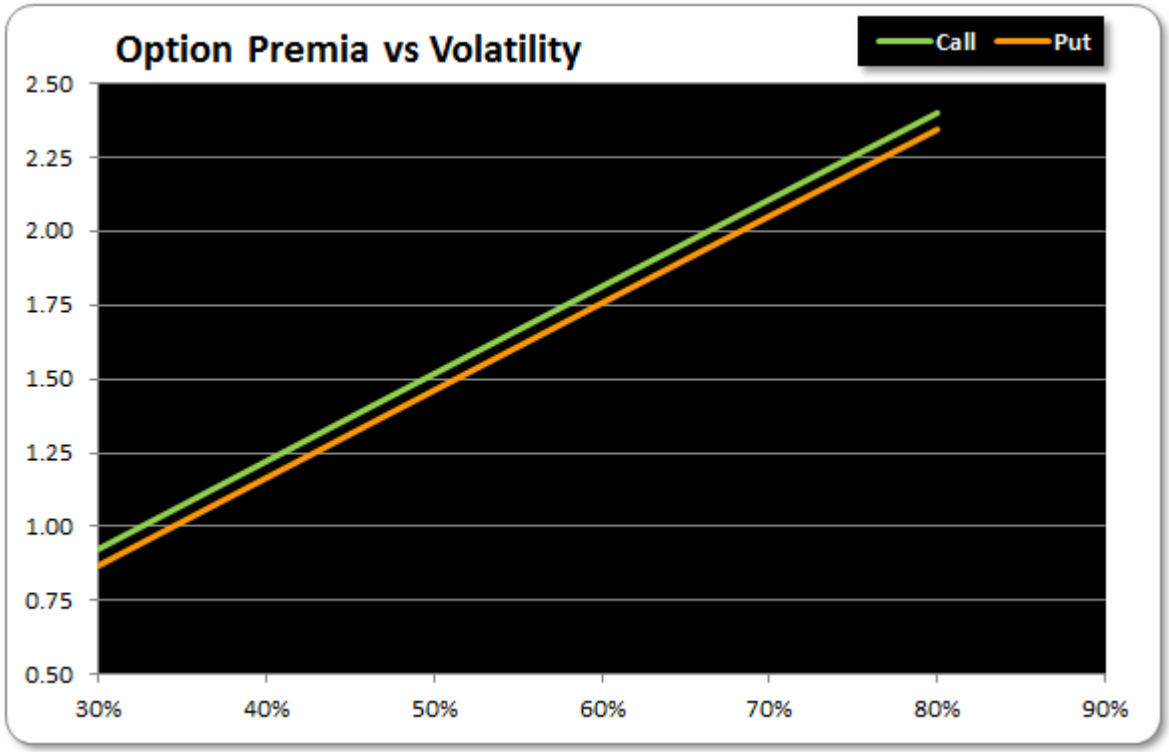
TANAY

Intraday swings in volatility could cause huge swings in value of derivatives

FUZZY LOGIX

Option Parameters	
Option Type	C
Strike Price	15.00
Spot Price	15.00
Time to Expiration (in years)	0.25
Risk-free Rate	1.5%

Volatility	Call Premium	Put Premium
30%	0.92	0.87
35%	1.07	1.02
40%	1.22	1.16
45%	1.37	1.31
50%	1.52	1.46
55%	1.67	1.61
60%	1.81	1.76
65%	1.96	1.90
70%	2.11	2.05
75%	2.25	2.20
80%	2.40	2.35

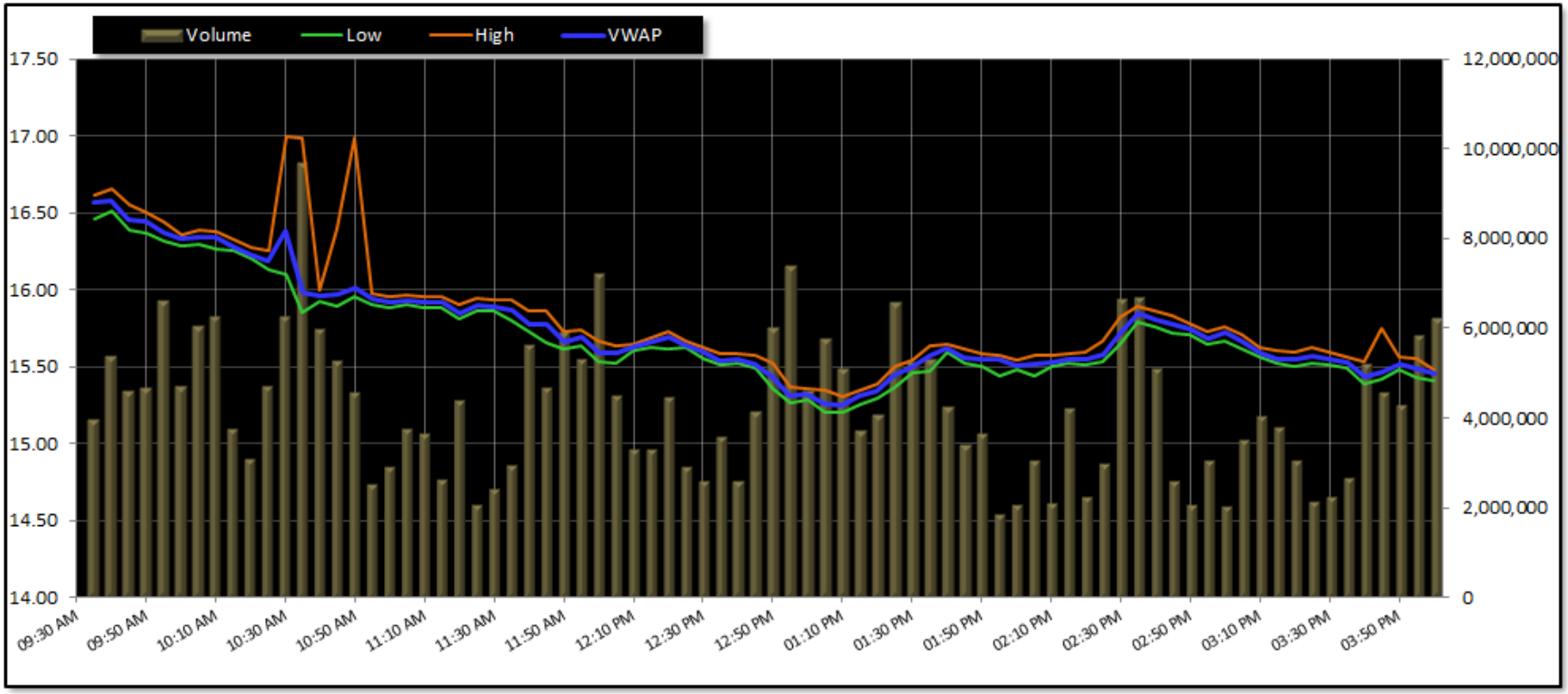


TANAY

High volatility is a result of wild swings in prices of underlying

FUZZY LOGIX

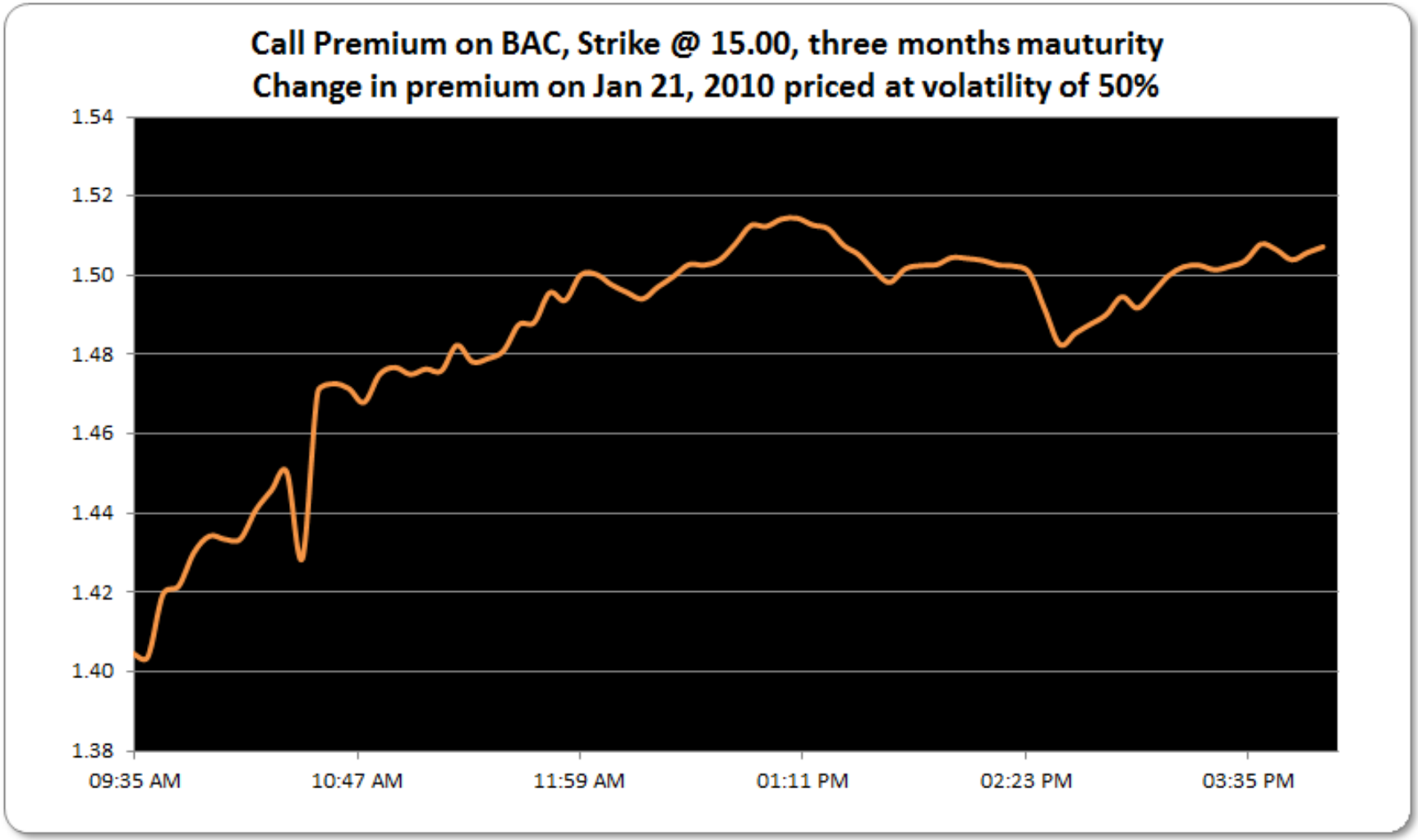
Volume Weighted Average Price (VWAP) for Bank of America on Jan 21, 2010
Almost a 7% loss for the day



TANAY

As a result, there are wild swings in prices of derivatives

FUZZY LOGIX



TANAY

Market Risk Management

The Contemporary Requirements

FUZZY LOGIX

- End-of-day calculations are inadequate – need to ensure that portfolios are within prescribed limits more frequently
- **Ensure compliance with margin limits and collateral requirements**
  - Institutional investors frequently trade large volumes on margin accounts and on accounts secured by collateral – market volatility could easily lead to violations in limits
- **Challenges with intra-day risk management**
  - Technology required for very high-throughput calculations
  - High-performance computations to be performed continuously throughout trading sessions

TANAY

Market Risk Management

The Contemporary Scenario

FUZZY LOGIX

- One way people are dealing with this challenge – set conservative limits so that the likelihood breaking through the margins is minimized.
- When markets are tame, these limits are often relaxed.
- **Impact of such practice:**
  - ***Prevent full use of margin and lead to loss of trading revenue for exchanges and broker-dealers***
  - ***Subjective relaxation of margin and collateral limits often leads to large losses when the tempest transitions from the teapot to the market***

TANAY

Intra-Day Risk Management

Computational Challenges

FUZZY LOGIX

- **Computational challenge with intra-day risk management**
  - Perform valuation of a large number of securities across a large number of simulated scenarios and quickly evaluate the Value-at-Risk (VaR)

Intra-day risk factors for Equity

Intra-day Risk Factor	Number of Scenarios
Spot Price	1,000
Risk Free Rate	1,000
Volatility	1,000
Total Scenarios	1,000,000,000

Intra-day risk factors for FX

Intra-day Risk Factor	Number of Scenarios
Spot Price	100
Domestic Risk Free Rate	100
Foreign Risk Free Rate	100
Volatility	100
Total Scenarios	100,000,000

TANAY

Intra-Day Risk Management

Parallelized Algorithms on GPU

FUZZY LOGIX

- **How has this been achieved?**
  - ***Combination matrix of intra-day risk factors are not created.***
  - Instead, we move the three small arrays of intra-day scenarios to GPU – spot prices, risk-free rates, volatility.
  - On the GPU thread, we can identify the particular combination for which the calculation is to be done.
  - Very small amount of data moved to GPU.

```
//Calculate the thread index
vBlockIndex = blockIdx.y * gridDim.x + blockIdx.x;
vThreadIndex = vBlockIndex * blockDim.x + threadIdx.x;

//Calculate the indexes
vSpotPriceIndex = (int)((vThreadIndex - 1)/pNumOfRiskFreeRates) + 1;
vRiskFreeRateIndex = vThreadIndex - (vSpotPriceIndex - 1) *
                     pNumOfRiskFreeRates;
```

TANAY

Intra-Day Risk Management

Parallelized Algorithms on GPU

FUZZY LOGIX

- **How has this been achieved?**
  - ***Hybrid parallel programming on GPU – data parallel and process parallel***
  - On each GPU thread, we can perform full valuation for all the scenarios for one factor keeping the other two fixed.

Spot Price	Volatility	Risk-free Rate		Spot Price	Volatility	Risk-free Rate					
15.00	35%	2.93%	Thread 1	15.25	36%	2.93%	Thread 2	16.93	29%	2.93%	Thread 3
		2.09%				2.09%				2.09%	
		3.93%				3.93%				3.93%	
		2.22%				2.22%				2.22%	
		3.01%				3.01%				3.01%	
		3.19%				3.19%				3.19%	
		3.17%				3.17%				3.17%	
		1.74%				1.74%				1.74%	
		3.56%				3.56%				3.56%	
		3.35%				3.35%				3.35%	

TANAY

Intra-Day Risk Management
Parallelized Algorithms on GPU

FUZZY LOGIX

- **How has this been achieved?**
  - ***Full valuation is expensive because spot price, risk-free rate and volatility is different on each thread. Avoiding full valuation is important.***

$$c = Se^{-ft}N(d1) - Xe^{-rt}N(d2)$$

$$p = -Se^{-ft}N(-d1) + Xe^{-rt}N(-d2)$$

Where
c = call premium
p = put premium
S = spot price
X = strike price
r = domestic risk free rate
f = foreign risk free rate
t = time to expiration
N(x) is the Cumulative Distribution Function (CDF) for the standard normal distribution for the range [-∞, x].

$$N(x) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{x} e^{\frac{-t^2}{2}}\, dt$$

$$d1 = \frac{ln\left(\frac{S}{X}\right) + \left(r - f + \frac{\sigma^2}{2}\right)t}{\sqrt{t}}$$

$$d2 = d1 - \sqrt{t}$$

σ = annualized volatility

TANAY

Intra-Day Risk Management

Parallelized Algorithms on GPU

FUZZY LOGIX

- **How has this been achieved?**
  - *Hybrid calculation of VaR*
    - *Calculate Rho on each thread and find the region of interest.*
    - *Perform full valuation only for the region of interest.*

Spot Price	Volatility	Risk-free Rate		Spot Price	Volatility	Risk-free Rate					
15.00	35%	1.42%	Thread 1	15.25	36%	1.42%	Thread 2	16.93	29%	1.42%	Thread 3
		1.40%				1.40%				1.40%	
		2.47%				2.47%				2.47%	
		1.56%				1.56%				1.56%	
		3.27%				3.27%				3.27%	
		1.84%				1.84%				1.84%	
		1.31%				1.31%				1.31%	
		1.38%				1.38%				1.38%	
		1.78%				1.78%				1.78%	
		2.21%				2.21%				2.21%	

TANAY

Intra-Day Risk Management

Parallelized Algorithms on GPU

FUZZY LOGIX

- **How has this been achieved?**
  - Intelligent indexing using GPU threads
  - Map reduce or parallel programming on GPU
  - Steepest descent using Greeks to find region of scenarios where VaR for a security is likely
  - Combination of steepest descent and full valuation executed in parallel in GPUs
  - ***In short using both data parallel and process parallel design***

TANAY

Intra-Day Risk Management

Equity Options & FX Options

FUZZY LOGIX

Performance Benchmarks

Number of Securities	Scenarios	Time Taken	Number of Full Valuations	Number of GPU Cards Used
1	1,000,000,000	50-60 milliseconds	1,000,000,000	1
100,000	100,000,000	125 seconds	10,000,000,000,000	8
1,000,000	100,000,000	300 seconds	100,000,000,000,000	32

Is that good performance?

TANAY

Computational Challenges for Financial Institutions

FUZZY LOGIX

Risk Management

- Intra-day Risk Management
- Closeout Risk & Margin Management

Pricing

- Publishing Intra-day prices for settlement
- Calibration to market
- Pricing of exotics and structured products

Compliance

- Real Time Compliance
- Anti Money Laundering

Large amount of complex computations required in milliseconds

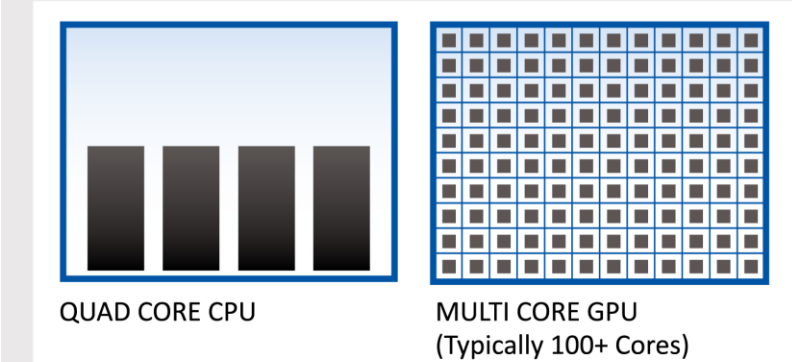
The solution lies in GPU-based analytics

TANAY

GPUs In Computing

FUZZY LOGIX

- GPUs were developed to accelerate graphics
- Over time, technology for GPUs has evolved. Today, GPUs can perform complex mathematical operations
- GPUs cannot do everything that CPUs can; GPUs have to work under the control of CPUs.
- The idea behind GPU – perform millions of mathematical operations in parallel on multiple cores at blazing speed



TANAY

Comparative Benchmarks

FUZZY LOGIX

Algorithm	Speed on Tesla C-2070 vs. Intel Xeon 8 core CPU
Monte Carlo Simulation – Normal, Log Normal, Student's T, Skewed Student's T, Beta, Gamma, Inverse Gamma	100 – 250 X faster
Black-Scholes Option Pricing	500-1000 X faster
Garman-Kohlhagen Model for Foreign Exchange Derivatives	300-750 X faster
Black's Model for Interest Rate Derivatives	500-1000 X faster
Pricing of exotics/structured products – Basket Options, Asian Options, Bermudans, Barrier Options	150-250 X faster

For all the options traded in US in a given day as reported by OPRA (500K to 1 million trades), implied volatility can be calculated by Tanay ZX Series in less than 500 milliseconds

TANAY

Comparative Benchmarks

FUZZY LOGIX

- For Monte Carlo Simulations, 100 million – 10 billion simulations were run for each distribution.
- For equity, FX and interest rate derivatives, three different types of calculations were performed – pricing of options, calculation of Greeks and calculation of implied volatility. Each of these calculations were performed 100 million times.
- Each of the exotic options i.e., basket options, Asian options, Bermudans and barrier options were priced using 1 million Monte Carlo simulations for the underlying.
- In both the cases, GPU and CPU, the code that was used has been developed by Fuzzy Logix. In all of these examples, the CPU code has been parallelized on GPU thus resulting in huge performance gains.
- The configuration of the server used for benchmarking is as follows:
  - CPU: Intel Xeon 2.66 GHz 8 Core
  - RAM: 24 GB DDR3 1333 MHz
  - HDD: 3.5” 1 TB SATA

TANAY

Contact Us

FUZZY LOGIX

Please visit us in the exhibition hall for discussions

Cyrus Golkar
Vice President – Business Development
Email – cyrus.golkar@fuzzyl.com
Phone – 408-858-7979

Jim Zhang
Chief Analytics Officer
Email - jim.zhang@fuzzyl.com
Phone – 704-236-6357

TANAY