Message Passing Parallelism for Belief Propagation in Junction Trees

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Motivation

Our Concern
Belief propagation over junction tree is computationally intensive.
- The topology and connectedness of BN
- The high cardinality of a significant set of discrete BN nodes

Observations
- In each message passing, computations associated with different separator table cells are independent.
- Some junction tree consists of large cliques and separators.

Therefore, Compute Each Message in Parallel
- Substantial parallel opportunity when neighboring cliques and separators are large.
- Non-invasively embedded in the original algorithm

Analysis of Speedup

Theoretically, the speedup is decided by the parallelism and GPU kernel invocation overhead.

\[
Speedup = \frac{\sum \left( \frac{2(n-1)\tau}{\log(N_h)} + \frac{2(n-1)\tau}{\log(N_b)} \right)}{\sum \left( \frac{2(n-1)\tau}{\log(N_h)} + \frac{2(n-1)\tau}{\log(N_b)} \right)}
\]

Parallel Message Passing

Cluster-Sepset Mapping parallelism

Arithmetic parallelism

Parallel Message Passing

Problem with Small Cliques

- Not enough opportunity for either kind of parallelism

Solution

- Merge Small cliques into a larger clique

Clique Merging

Observations

- In each message passing, computations associated with different separator table cells are independent.
- Some Junction tree consists of large cliques and separators.

Experimental Results

Cross platform comparison result

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Pigs</th>
<th>Mini2</th>
<th>Mini2</th>
<th>Mini3</th>
<th>Midden</th>
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<th>Barley</th>
<th>Barley</th>
<th>Diabetes</th>
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<tbody>
<tr>
<td># of original J nodes</td>
<td>366</td>
<td>666</td>
<td>994</td>
<td>972</td>
<td>28</td>
<td>20</td>
<td>56</td>
<td>337</td>
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<tr>
<td># of JI nodes after merge</td>
<td>162</td>
<td>553</td>
<td>653</td>
<td>564</td>
<td>22</td>
<td>18</td>
<td>35</td>
<td>334</td>
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<tr>
<td>Ave. CPT size in original JI</td>
<td>1.972</td>
<td>5.655</td>
<td>3.443</td>
<td>16.444</td>
<td>341.651</td>
<td>173.297</td>
<td>512.644</td>
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<td>5.923</td>
<td>40.400</td>
<td>74.999</td>
<td>141.008</td>
<td>411.319</td>
<td>163.333</td>
<td>81.82</td>
<td>68.26</td>
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<td>Ave. SPT size in original JI</td>
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<td>533</td>
<td>2,099</td>
<td>9,273</td>
<td>26,065</td>
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<td>GPU time before merge (ms)</td>
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<td>98.45</td>
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<td>GPU time after merge (ms)</td>
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<td>Merging Speedup</td>
<td>1.36X</td>
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<td>1.14X</td>
<td>1.10X</td>
<td>1.02X</td>
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<td>1.01X</td>
<td>1.02X</td>
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<td>CPU time (s)</td>
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<td>240</td>
<td>137</td>
<td>473</td>
<td>355</td>
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<td>974</td>
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<td>Cross platform speedup</td>
<td>2.20X</td>
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</table>

References


Platform

- NVIDIA GeForce GTX460
  - 48 cores
  - 48KB on-chip shared memory
  - Memory Bandwidth 90GB/s
- Peak CUDA

V.S.

- Intel Core2 Quad CPU
  - 4 cores
  - 2.5GHz processor clock
  - 8MB cache
  - 9 GB memory
  - C++

Conclusion & Future Work

- A novel approach to parallel belief propagation over junction trees, based on the cluster-sepset mapping method.
- Analysis of speedup as a function of junction tree and GPU parameters.
- A novel clique merging technique to make better use of the parallelism provided by GPU.
- Experiments show the approach works well when junction trees have few but large cliques and separators.